Today we'll pick up where we left off on Tuesday. We'll discuss a graphical solution to the 3-point problem and then discuss basic vector concepts with applications to stream flow and fault offset. We'll wrap up with some in-class problems. Next week will be devoted entirely to review.
1. The 180° rule: \( A + B + C = 180 \)

2. \[
\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)} \quad \text{Sine rule}
\]

3. \[
a^2 = b^2 + c^2 - 2bc \cdot \cos(A) \quad \text{Cosine rule}
\]

\[
A = \cos^{-1} \left[ \frac{b^2 + c^2 - a^2}{2bc} \right]
\]

Rather than approximate as we did on Tuesday, what approach can we use to arrive at a more exact solution?

\[
\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)} \quad \frac{3.26}{\sin(45)} = \frac{b}{\sin(85)} = \frac{c}{\sin(50)}
\]
3.26 km C to T
4.6 km C to E
3.26 km T to E

Similarly you’ll find that c = 3.53 km, so that our right angle approximation gives us about 8% error.

\[
\sin(45) \sin(85) \sin(50) = \frac{3.26}{\sin(45)} = 4.61
\]

\[
\frac{b}{\sin(85)} = \frac{b}{0.996} \Rightarrow b = 0.996 \times 4.61 = 4.59 \text{ km C to E}
\]

What angles do the points A-D make with North?

For point A - What is the tangent of the angle you are interested in?

\[
\tan \delta = \frac{5}{10} = 0.5
\]

\[
\arctan(0.5) = 26.6^\circ
\]

What is the angle between OA and OB?

What is the angle a line drawn to point B makes with north?
Question 1: How does the velocity at point B differ from the velocity at point A?

Question 2: Does the vector sum of C and D equal B?
If not, how has the velocity changed?

What geological processes are accommodated by this change.

See [http://mathforum.org/~klotz/Vectors/](http://mathforum.org/~klotz/Vectors/)

find an excel file titled vector.xls on the common drive or accessible from the class website. Open up and have a look as we review some vector computation ideas.

\[ \vec{A} = x_A \hat{i} + y_A \hat{j} \]
\[ \vec{B} = x_B \hat{i} + y_B \hat{j} \]

**Analysis of Figure 4.34 (Klotz)**

<table>
<thead>
<tr>
<th>Location</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow Speed</td>
<td>170</td>
<td>150</td>
<td>160</td>
<td>250</td>
</tr>
<tr>
<td>East-West Component</td>
<td>0.0824</td>
<td>-1.0110</td>
<td>1.0106</td>
<td>0.0804</td>
</tr>
<tr>
<td>North-South Component</td>
<td>-4.2419</td>
<td>2.5030</td>
<td>2.4150</td>
<td>3.8732</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>A · B</th>
<th>E-W Reagent</th>
<th>N-S Reagent</th>
<th>Reagent Velocity</th>
<th>Flow Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-1.9013</td>
<td>3.5140</td>
<td>2.199350</td>
<td>27.2744</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>C · D</th>
<th>E-W Reagent</th>
<th>N-S Reagent</th>
<th>Reagent Velocity</th>
<th>Flow Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-5.4024</td>
<td>-2.9504</td>
<td>4.730855</td>
<td>85.2046</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B · (C-D)</th>
<th>E-W Reagent</th>
<th>N-S Reagent</th>
<th>Reagent Velocity</th>
<th>Flow Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-1.3531</td>
<td>-1.2106</td>
<td>1.389151</td>
<td>41.3733</td>
</tr>
</tbody>
</table>
Notice that the cells to the left of “East-West Component” have cell formulae that look something like =C4*SIN(C5*(PI()/180)). What are we doing here?

Notice that the cells to the left of “North-South Component” have cell formulae that look something like =C4*COS(C5*(PI()/180)).
Examine the results. What do they tell you?

The magnitude of the change is about 2.15 m/s. The vector is oriented 27.3 degrees below the EW axis in the 3rd quadrant.
Flow speed has increased and the flow direction has shifted to the southwest.

Now consider whether the vector sum of C and D equal B?

How has the velocity changed? What geological processes are accommodated by this change?

Note that the resultant velocity of C+D (4.7 m/s) is less than that at B (6 m/s). The resultant vector difference ((C+D)-B) points into the northeast quadrant.
Let's draw these two vectors and think about how things have changed.

B has magnitude of 6 m/s

C+D
Magnitude
4.71 m/s

Resultant
B – (C+D)

We need to use common sense to figure out how things have changed. The change is one relative to B. The minus signs actually indicate decrease in the north and east directions relative to B.
Which bank of the stream at the fork experiences the greatest erosion?

Net flow associated defined by the resultant flow \((C+D)\) is deflected to the northeast. One could reason that the northeast facing bank at the fork will be preferentially eroded.

Since channel D lies nearly parallel to the B, there may be other factors such as channel depth that influence resultant flow velocity in channels D and C.

In the Excel file Vector.xls, click on the **Net Fault Slip** worksheet to open it.

You’ll notice that the horizontal and vertical displacements for Fault B (the one to the right in the above diagram) have not been computed. Also, the total combined slip and equivalent fault dip (or net slip dip) have also not been computed. Take a moment and make those computations in your excel spreadsheet.
Net slip along fault
\[ B = 13.5 \text{ m} \]
\[ \theta = 38^\circ \]
Horizontal displacement = 13.5 cos(38)
Vertical Displacement = 13.5 sin(38)

Cumulative Horizontal Displacement (=h)
Cumulative Vertical Displacement (=v)

Cumulative
Vertical
Displacement
Cumulative
Horizontal Displacement

Dip = ?

Total combined slip = \[ \sqrt{h^2 + v^2} \]

\[ h = \arctan(v/h) \]
Look at the formula in cells C5 and C6 (=C3*COS(C4*(PI()/180)) and =C3*SIN(C4*(PI()/180)), respectively). Make sure you understand what is being computed.

The total combined slip is the length or magnitude of the resultant vector.

The equivalent fault dip is the dip a single fault would have in order to produce the identical net slip. This is just the dip of the combined slip vector. How do you compute this?

<table>
<thead>
<tr>
<th>Fault</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net Slip</td>
<td>18.8</td>
<td>18.5</td>
</tr>
<tr>
<td>Flow Direction</td>
<td>46</td>
<td>33</td>
</tr>
<tr>
<td>Horizontal Displacement</td>
<td>13.6596</td>
<td>10.630</td>
</tr>
<tr>
<td>Vertical Displacement</td>
<td>13.5236</td>
<td>8.3114</td>
</tr>
</tbody>
</table>

$A = B$

Cumulative Horizontal Displacement | 23.6977 |
Cumulative Vertical Displacement  | 21.835  |
Total Combined Slip               | 32.2234 |
Equivalent Single Fault Dip      | 42.6574 |

The following problems from Chapter 5 will be discussed in class during review week: 5.19, 5.20, 5.21, and 5.25.

These problems along with today’s in-class problems will provide opportunities to prepare for the final.

The final will focus on materials covered in chapters 5, 8, and 9.
The following slides are presented as a supplement to the discussions in the text. We will not cover in class.

\[
\tan(\theta) = \frac{OA}{OB} \quad \tan(\theta') = \frac{OA}{OC} \quad \cos(\alpha) = \frac{OB}{OC}
\]

\[
\tan(\theta') = \frac{OA}{OB} \cdot \frac{OB}{OC} = \tan(\theta) \cos(\alpha)
\]

Question 5.12: On a cliff face, the apparent dip is 25° while the true dip is 35°. What is the angle between the cliff face and the strike direction?

\[
\tan(\theta') = \tan(\theta) \cos(\alpha)
\]
Given
\[ \tan(\theta') = \tan(\theta) \cos(\alpha) \]

This implies
\[ \theta' = \arctan(\tan(\theta) \cos(\alpha)) \] (i.e. \( \arctan = \text{arc tangent or } \tan^{-1} \))

\[ \theta = \arctan(\tan(\theta') \cos(\alpha)) \] or
\[ \alpha = \cos(\tan(\theta'/\tan(\theta)) \]

\[ \alpha = \cos(\tan(25^\circ)/\tan(35^\circ)) \]
\[ \alpha = \cos(0.466/0.7) \]
\[ \alpha = \cos(0.667) \]
\[ \alpha = 48^\circ \]