In geology we’re usually trying to come up with a mathematical model that explains or characterizes a geologic process. One equation rarely explains the variety of behaviors associated with even a single geological process. And one way or another we are always making some assumptions in the approach we choose to estimate things.
This is a quadratic equation. The general form of a quadratic equation is

\[ y = ax^2 + bx + c \]

![Quadratics graph]

One equation differs form that used in the text

We see that the variations of T with Depth are nearly linear in certain regions of the subsurface. In the upper 100 km the relationship \( T = 20x + 10 \) provides a good approximation.

From 100-700km the relationship \( T = 1.25x + 1017 \) works well.

Can we come up with an equation that will fit the variations of temperature with depth - for all depths?

Let's try a quadratic.
The quadratic relationship plotted below is just one possible relationship that could be derived to explain the temperature depth variations.

\[ T = (1.537 \times 10^{-4}) x^2 + 1.528 x + 679.77 \]

These are members of the general class of functions referred to as polynomials. A polynomial is an equation that includes \( x \) to the power 0, 1, 2, 3, etc. The straight line

\[ y = mx + b \]

is referred to as a first order polynomial. The order corresponds to the highest power of \( x \) present in the equation - in the above case the highest power is 1.

The quadratic \( y = ax^2 + bx + c \) is a second order Polynomial, and the equation

\[ y = ax^n + bx^{n-1} + cx^{n-2} + \ldots + a_0 \]

is an nth order polynomial.
Can we obtain more accurate representations?

Here is another 4th order polynomial which in this case attempts to fit the near-surface 100km. Notice that this 4th order equation (redline plotted in graph) has three bends or turns.

\[ T = -1.289 \times 10^{-11} d^4 + 1.99 \times 10^{-7} x^3 - 0.00113 x^2 + 3.054 x + 394.41 \]
In sections 2.5 and 2.6 Waltham reviews negative and fractional powers. The graph below illustrates the set of curves that result as the exponent $p$ in $y = ax^p + c$ is varied from 2 to -2 in -0.25 steps, and $c$ equals 0. Note that the negative powers rise quickly up along the y axis for values of $x$ less than 1 and that $y$ rises quickly with increasing $x$ for $p$ greater than 1.

2\(^2\) = 4
What is 0.01\(^2\)?
What is 0.01\(^{-2}\)?

Power Laws - A power law relationship relevant to geology describes the variations of ocean floor depth as a function of distance from a spreading ridge ($x$).

$$d = ax^{1/2} + d_0$$

What physical process do you think might be responsible for this pattern of seafloor subsidence away from the spreading ridges?
Section 2.7 Allometric Growth and Exponential Functions

Allometric - differential rates of growth of two measurable quantities or attributes, such as Y and X, related through the equation $Y=ab^X$.

Examples: Age vs. Depth, radioactive decay, earthquake frequency and magnitude, porosity and depth relationships …

This relationship is not linear. A straight line does a poor job of passing through the data points. The slope (gradient or rate of change) decreases with increased depth.

Just as an aside, what does the relationship $A=kZ$ imply about porosity variation as a function of depth?

Draw a graph

Waltham generates this data using the following relationship.

$$\phi = 0.6 \times 2^{-z}$$
These compaction effects make the age-depth relationship non-linear. The same interval of depth $\Delta D$ at large depths will include sediments deposited over a much longer period of time than will a shallower interval of the same thickness.

Porosity might vary with depth more like that shown in the table below -

<table>
<thead>
<tr>
<th>Depth (km)</th>
<th>Porosity ($\phi$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.6</td>
</tr>
<tr>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td>2</td>
<td>0.15</td>
</tr>
<tr>
<td>3</td>
<td>0.075</td>
</tr>
<tr>
<td>4</td>
<td>0.0375</td>
</tr>
</tbody>
</table>

Over the range of depth 0-4 km, the porosity decreases from 60% to 3.75%! 
\[ \phi = 0.6 \times 2^{-z} \]

This equation assumes that the initial porosity (0.6) decreases by 1/2 from one kilometer of depth to the next. Thus the porosity (\(\phi\)) at 1 kilometer is \(2^{-1}\) or 1/2 that at the surface (i.e. 0.3), \(\phi(2)=1/2 \times \phi(1)=0.15\) (i.e. \(\phi=0.6 \times 2^{-2}\) or 1/4th of the initial porosity of 0.6.

Equations of the type

\[ y = ab^{cx} \]

are referred to as allometric growth laws or exponential functions.

In the equation \(\phi = 0.6 \times 2^{-z}\)

- \(a = ?\) \(a=0.6\)
- \(b = ?\) \(b=2\)
- \(c = ?\) \(c=-1\)

The constant \(b\) is referred to as the base.

Recall that in relationships like \(y=10^a\), \(a\) is the power to which the base 10 is raised in order to get \(y\).
Logarithms

Above, when we talked about functions like
\[ y = ab^{cx} \]
and
\[ y = a10^{cx} \]
b and 10 are what we refer to as bases. These are constants and we can define any other number in terms of these constants raised to a certain power.

Given any number \( y \), we can express \( y \) as 10 raised to some power \( x \)

\[ i.e. \ y = 10^x \]

Thus, given \( y = 100 \), we know that \( x \) must be equal to 2.

\[ y = 10^x \]

By definition, we also say that \( x \) is the log of \( y \), and can write

\[ \log y = \log(10^x) = x \]

So the powers of the base are logs. “log” can be thought of as an operator like \( x \) and \( ÷ \) which yields a certain result. Unless otherwise noted, the operator “log” is assumed to represent log base 10. So when asked what is

\[ \log y \text{, where } y = 45 \]

We assume that we are asking for \( x \) such that

\[ 10^x = 45 \]
One of the most commonly used “Richter magnitude” scales determines the magnitude of shallow earthquakes from surface waves according to the following equation

\[ m = \log_{10} \frac{A}{T} + 1.66 \log \Delta + 3.3 \]

where \( T \) is the period in seconds, \( A \) the maximum amplitude of ground motion in \( \mu m \) (\( 10^{-6} \) meters) and \( \Delta \) is the epicentral distance in degrees between the earthquake and the observation point.

We’ve already worked with three bases - 2, 10 and e. Whatever the base, the logging operation is the same. \( \log_5 10 \) asks what is the power that 5 must be raised to to get 10.

\[ \log_5 10 = ? \]

**How do we find these powers?**

\[ \log_5 10 = \frac{\log_{10} 10}{\log_{10} 5} \]

\[ \log_5 10 = \frac{1}{0.699} = 1.431 \]

Thus \( 5^{1.431} = 10 \)
In general,  \[ \log_{base} (some\ number) = \frac{\log_{10}(number)}{\log_{10} base} \]

or  \[ \log_{b} a = \frac{\log_{10}(a)}{\log_{10} b} \]

Try the following on your own

\[ \log_{3} 7 = \frac{\log_{10}(7)}{\log_{10} 3} = ? \]

\[ \log_{8} 8 \]

\[ \log_{7} 21 \]

\[ \log_{4} 7 \]

As noted above  \( \log_{10} \) is often written as  \( \log \), with no subscript.

\( \log_{10} \) is referred to as the common logarithm

\( \log_{e} \) is often written as  \( \ln \).

thus  \[ \log_{e} 8 = \ln 8 = 2.079 \]

\( \log_{e} \) or \( \ln \) is referred to as the natural logarithm. All other bases are usually specified by a subscript on the log, e.g.

\( \log_{5} \) or \( \log_{2} \), etc.
Finish reading Chapters 1 and 2 (pages 1 through 38) of Waltham and begin reading through Chapter 3.

After we finish our basic review, we will learn how to use Excel a scientific computing and graphing software package to solve some problems related to the materials we’ve just covered in Chapters 1 and 2.