Let's take a short tour of the Excel files provided by Waltham that are coordinated with text exercises and discussions.

Click on this link: Take a look at the following:

- Intro.xls
- Exp.xls
- Integ.xls
Warm-up Exercise: Part 1

1. What is the difference in length between a 12-inch and a 18-inch object?
   a) 6 inches
   b) 12 inches
   c) 18 inches
   d) 24 inches
   e) 30 inches

2. A formula relates the Fahrenheit (F) and Celsius (C) scales as given by: \( F = \frac{9}{5}C + 32 \). How many degrees Fahrenheit is 0 degrees Celsius?
   a) 0°F
   b) 10°F
   c) 20°F
   d) 30°F
   e) 40°F

3. What is the area of a square whose area is 64 square inches?
   a) 8
   b) 16
   c) 32
   d) 64
   e) 128

4. What is the area of a square foot of the figure at right?
   a) 12
   b) 30
   c) 42
   d) 63
   e) 60

5. A formula relates the Fahrenheit (F) and Celsius (C) scales as given by: \( F = \frac{9}{5}C + 32 \). What is the area of a square foot of the figure at right?

6. What is the perimeter in inches of a square whose area is 64 square inches?
   a) 8
   b) 16
   c) 32
   d) 64
   e) 128

10. In the graph below, porosity is plotted as a function of depth in kilometers. What is the average porosity between depths of 2 and 5 kilometers?

   a) 0.11
   b) 0.13
   c) 0.15
   d) 0.14
   e) 0.2

![Graph of porosity vs. depth]
Subscripts and Superscripts

Subscripts and superscripts provide information about specific variables and define mathematical operations.

$k_1$ and $k_2$ for example could be used to denote sedimentation constants for different areas or permeabilities of different rock specimens.

See Waltham for additional examples of subscript notation.

Superscripts - powers

The geologist’s use of math often turns out to be a periodic and necessary endeavor. As time goes by you may find yourself scratching your head pondering once-mastered concepts that you suddenly find a need for.

This is often the fate of basic power rules.

*Evaluate the following*

\[ x^a x^b = x^{a+b} \]
\[ x^a / x^b = x^{a-b} \]
\[ (x^a)^b = x^{ab} \]
Question 1.2a Simplify and where possible evaluate the following expressions -

i) $3^2 \times 3^4$

ii) $(4^2)^{2+2}$

iii) $g^i \cdot g^k$

iv) $D^{1.5} \cdot D^2$
Exponential notation is a useful way to represent really big numbers in a small space and also for making rapid computations involving large numbers - for example,

- the mass of the earth is \( 597 \times 10^{22} \text{ kg} \)
- the mass of the moon is \( 7.35 \times 10^{22} \text{ kg} \)

Express the mass of the earth in terms of the lunar mass.

While you’re working through that with pencil and paper let me write down these two masses in exponential notation.

\[
M_E = 597 \times 10^{22} \text{ kg} \\
M_M = 7.35 \times 10^{22} \text{ kg}
\]

The mass of the moon \((M_M)\) can also be written as \(0.0735 \times 10^{24}\text{ kg}\)

Hence, the mass of the earth expressed as an equivalent number of lunar masses is

\[
M_{E (M)} = \frac{M_E}{M_M} = \frac{597 \times 10^{22}}{7.35 \times 10^{22}} = \frac{597}{7.35} = 81.2\text{ lunar masses}
\]
Write the following numbers in exponential notation (powers of 10)?

The mass of the earth’s crust is 280 000 000 000 000 000 000 000 kg
The volume of the earth’s crust is 1 000 000 000 000 000 000 000 m³

What is the density of the earth’s crust?

The mass of the earth’s crust is $2.8 \times 10^{22}$ kg
The volume of the earth’s crust is $1 \times 10^{19}$ m³

$= \frac{\text{mass}}{\text{volume}} = 2.8 \times 10^3$ kg/m³

Differences in the acceleration of gravity on the earth’s surface (and elsewhere) are often reported in milligals. 1 milligal = $10^{-5}$ meters/second².

What is $9.8$ m/s² in milligals?

This is basically a unit conversion problem - you are given a value in one system of units, and the relation of requested units (in this case milligals) to the given units (in this case meters/s²)

1 milligal = $10^{-5}$ m/s² hence 1 m/s² in terms of milligals is found by multiplying both sides of the above equation by $10^5$

to yield $10^5$ milligals = 1 m/s² - thus

$g = (9.8$m/s²$) \times 10^5$ milligals/(m/s²) = $9.8 \times 10^5$ milligals
The earth gains mass every day due to collision with (mostly very small) meteoroids. Estimate the increase in the earth’s mass since its formation assuming that the rate of collision has been constant and that

\[ \frac{\Delta M}{\Delta t} = 6 \times 10^5 \text{ kg/day} \]

\[ A_e = 4.5 \times 10^9 \text{ years} \]

\[ \Delta M/\Delta t \text{ is the rate of mass gain} \]

\[ A_e \text{ is the age of the earth (our } \Delta t) \]

But, what is the age of the earth in days?

\[ A_E = 365 \frac{\text{days}}{\text{year}} \times 4.5 \times 10^9 \text{ years} \]

1) What is the total mass gained?

2) Express the mass-gain as a fraction of the earth’s present day mass

\[ i.e. A_E (\Delta M / \Delta t) \]

\[ M_E = 5.95 \times 10^{24} \text{ kg} \]

\[ A_E \text{ is a } \Delta t \]
The North Atlantic Ocean is getting wider at the average rate $v_s$ of $4 \times 10^{-2}$ m/y and has width $w$ of approximately $5 \times 10^6$ meters.

1. Write an expression giving the age, $A$, of the North Atlantic in terms of $v_s$ and $w$.

2. Evaluate your expression to answer the question - When did the North Atlantic begin to form?

Age versus depth relationship(s)

How thick was it originally?

Over what length of time was it deposited?
Consider another depositional environment
Any ideas where this is?

It's cold there

Troughs

Layered Deposits?
510,000 years

0.5 to 2.1 million years ago

2.1 to 2.7 million years ago

Variations induced by Astronomical Cycles?

Milankovitch Cycles

**ECCENTRICITY**

- MORE ELONGATED
- LESS ELONGATED

PERIODICITY: 100,000 YEARS

**AXIAL TILT**

PERIODICITY: 41,000 YEARS

**PRECESSION**

PERIODICITY: ~26,000 YEARS
Chapter 1
Mathematics as a tool for solving geological problems

The example presented on page 3 illustrates a simple age-depth relationship for un lithified sediments

\[ Age = k \times depth \]

This equation is a quantitative statement of what we all have an intuitive understanding of - increased depth of burial translates into increased age of sediments. But as Waltham suggests - this is an approximation of reality.

What does this equation assume about the burial process? Is it a good assumption?


http://www.sciencedaily.com/releases/2008/04/080420114718.htm
Example - if $k = 1500 \text{ years/m}$ calculate sediment age at depths of 1m, 2m and 5.3m. Repeat for $k = 3000 \text{ years/m}$

1m \quad Age = 1500 \text{ years}
2m \quad Age = 3000 \text{ years}
5.3m \quad Age = 7950 \text{ years}

For $k = 3000 \text{ years/m}$

Age = 3000 \text{ years}
Age = 6000 \text{ years}
Age = 15900 \text{ years}

**Symbolic notation** \rightarrow \text{age} = kz

*where*

\text{a}= \text{age}, \text{z}= \text{depth}

---

**Chapter 2 - Common relationships between geological variables**

You probably recognized that the equation we started with

$$Age = k \times Depth$$

*is the equation of a straight line.*

**The general equation of a straight line is**

$$y = mx + b$$
In this equation - \[ y = mx + b \]
which term is the slope and which is the intercept?

In this equation \[ Age = k \times \text{Depth} \]
which term is the slope and which is the intercept?

- \( k \) is the slope of the line
- the intercept must be zero

A more generalized representation of the age/depth relationship should include an intercept term - \[ A = kD + A_0 \]

The slope of the line is, in this case, an inverse rate. Our dependant variable is depth, which would have units of meters or feet, for example. The equation defines depth of burial in terms of age. \( k \), the slope transforms a depth into a number of years and \( k \) must have units of years/depth.

The geologic significance of \( A_0 \) - the intercept - could be associated with the age of the upper surface after a period of erosion. Hence the exposed surface of the sediment deposit would not be the result of recent sedimentation but instead would be the remains of sediments deposited at an earlier time \( A_0 \).
The slope of this line is $\Delta t/\Delta x = 1500\text{years/meter}$, what is the intercept?

The intercept is the line’s point of intersection along the y (or Age) axis at depth $=0$.

If only the relative ages of the sediments are known, then for a given value of $k$ (inverse deposition rate) we would have a family of possible lines defining age versus depth.

What are the intercepts?

Are all these curves realistic?
Consider the case for sediments actively deposited in a lake.

\[ A = kD + A_0 \]

Consider the significance of \( A_0 \) in the following context

If \( k \) is 1000 years/meter, what is the velocity that the lake bed moves up toward the surface?

If the lake is currently 15 meters deep, how long will it take to fill up?

\[ A = kD + A_0 \]

The slope \((k)\) does not change. We still assume that the thickness of the sediments continues to increase at the rate of 1 meter in 1000 years.

What is the intercept?

Hint: \( A \) must be zero when \( D \) is 15 meters
You should be able to show that $A_0$ is -15,000 years. That means it will take 15,000 years for the lake to fill up.

Our new equation looks like this -

$$A = 1000D - 15,000$$
... we would guess that the increased weight of the overburden would squeeze water from the formation and actually cause grains to be packed together more closely. Thus meter thick intervals would not correspond to the same interval of time. Meter-thick intervals at greater depths would correspond to greater intervals of time.

We might also guess that at greater and greater depths the grains themselves would deform in response to the large weight of the overburden pushing down on each grain.
These compaction effects make the age-depth relationship non-linear. The same interval of depth $\Delta D$ at large depths will include sediments deposited over a much longer period of time than will a shallower interval of the same thickness.

The relationship becomes non-linear.

The line $y=mx+b$ really isn’t a very good approximation of this age depth relationship. To characterize it more accurately we have to introduce non-linearity into the formulation. So let’s start looking at some non-linear functions.

Compare the functions

$A = 1000 \ D - 15,000$

and (in red)

$A = 3D^2 + 1000 \ D - 15,000$

What kind of equation is this?
This is a quadratic equation. The general form of a quadratic equation is

\[ y = ax^2 + bx + c \]

\[ y = 2x^2 + 10x + 20 \]
\[ y = 2x^2 \]
\[ y = 3x^2 - 60 \]

One equation differs from that used in the text

The increase of temperature with depth beneath the earth’s surface is a non-linear process.

Waltham presents the following table:

<table>
<thead>
<tr>
<th>Depth (km)</th>
<th>Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>100</td>
<td>1150</td>
</tr>
<tr>
<td>400</td>
<td>1500</td>
</tr>
<tr>
<td>700</td>
<td>1900</td>
</tr>
<tr>
<td>2800</td>
<td>3700</td>
</tr>
<tr>
<td>5100</td>
<td>4300</td>
</tr>
<tr>
<td>6360</td>
<td>4300</td>
</tr>
</tbody>
</table>
We see that the variations of T with Depth are nearly linear in certain regions of the subsurface. In the upper 100 km the relationship $T = 20x + 10$ provides a good approximation.

From 100-700km the relationship $T = 1.25x + 1017$ works well.

Can we come up with an equation that will fit the variations of temperature with depth - for all depths?

Let’s try a quadratic.

The quadratic relationship plotted below is just one possible relationship that could be derived to explain the temperature depth variations.

$T = (1.537 \times 10^{-4})x^2 + 1.528 x + 679.77$
The formula - below right - is presented by Waltham. In his estimate, he has not tried to replicate the variations of temperature in the upper 100km of the earth.

\[ T = (1.537 \times 10^{-4})x^2 + 1.53x + 680 \]

\[ T = (-8.255 \times 10^{-5})x^2 + 1.05x + 1110 \]

Either way, the quadratic approximations do a much better job than the linear ones, but, there is still significant error in the estimate of \( T \) for a given depth.

Can we do better?
To do so, we explore the general class of functions referred to as polynomials. A polynomial is an equation that includes x to the power 0, 1, 2, 3, etc. The straight line \( y = mx + b \) is referred to as a first order polynomial. The order corresponds to the highest power of x present in the equation - in the above case the highest power is 1.

The quadratic \( y = ax^2 + bx + c \) is a second order Polynomial, and the equation

\[
y = ax^n + bx^{n-1} + cx^{n-2} + \ldots + a_0
\]

is an nth order polynomial.

In general the order of the polynomial tells you that there are n-1 bends in the data or n-1 bends along the curve. The quadratic, for example is a second order polynomial and it has only one bend. But the number of bends in the data is not necessarily a good criteria for determining what order polynomial should be used to represent the data.
The temperature variations rise non-linearly toward a maximum value (there is one bend in the curve), however, the quadratic equation (second order polynomial) does not do an adequate job of defining these variations with depth.

Noting the number of bends in the curve might provide you with a good starting point. You could then increase the order to obtain further improvements.

Waltham offers the following 4th order polynomial as a better estimate of temperature variations with depth.

\[ T = -1.12 \times 10^{-12} d^4 + 2.85 \times 10^{-8} x^3 - 0.0031 x^2 + 1.64 x + 930 \]
Here is another 4th order polynomial which in this case attempts to fit the near-surface 100km. Notice that this 4th order equation (redline plotted in graph) has three bends or turns.

\[ T = -1.289 \times 10^{-11} d^4 + 1.99 \times 10^{-7} x^3 - 0.00113 x^2 + 3.054 x + 394.41 \]

In sections 2.5 and 2.6 Waltham reviews negative and fractional powers. The graph below illustrates the set of curves that result as the exponent \( p \) in

\[ y = a x^p + a_0 \]

is varied from 2 to -2 in -0.25 steps, and \( a_0 \) equals 0. Note that the negative powers rise quickly up along the y axis for values of \( x \) less than 1 and that \( y \) rises quickly with increasing \( x \) for \( p \) greater than 1.

\[ 2^2 = 4 \]
What is \( 0.01^2 \)?
What is \( 0.01^{-2} \)?
Power Laws - A power law relationship relevant to geology describes the variations of ocean floor depth as a function of distance from a spreading ridge (x).

\[ d = ax^{1/2} + d_0 \]

What physical process do you think might be responsible for this pattern of seafloor subsidence away from the spreading ridges?

Section 2.7 Allometric Growth and Exponential Functions

Allometric - differential rates of growth of two measurable quantities or attributes, such as Y and X, related through the equation \( Y = ab^x \).

This topic brings us back to the age/thickness relationship. Earlier we assumed that the length of time represented by a certain thickness of a rock unit, say 1 meter, was a constant for all depths. However, intuitively we argued that as a layer of sediment is buried it will be compacted - water will be squeezed out and the grains themselves may be deformed. The open space or porosity will decrease.
Waltham presents us with the following data table -

<table>
<thead>
<tr>
<th>Depth (km)</th>
<th>Porosity (φ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.6</td>
</tr>
<tr>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td>2</td>
<td>0.15</td>
</tr>
<tr>
<td>3</td>
<td>0.075</td>
</tr>
<tr>
<td>4</td>
<td>0.0375</td>
</tr>
</tbody>
</table>

Over the range of depth 0-4 km, the porosity decreases from 60% to 3.75%!

This relationship is not linear. A straight line does a poor job of passing through the data points. The slope (gradient or rate of change) decreases with increased depth.

Waltham generates this data using the following relationship.

\[ \phi = 0.6 \times 2^{-z} \]
This equation assumes that the initial porosity (0.6) decreases by 1/2 from one kilometer of depth to the next. Thus the porosity ($\phi$) at 1 kilometer is $2^{-1}$ or 1/2 that at the surface (i.e. 0.3), $\phi(2) = 1/2 \phi(1) = 0.15$ (i.e. $\phi = 0.6 \times 2^{-2}$ or 1/4th of the initial porosity of 0.6.

Equations of the type

$$y = ab^{cx}$$

are referred to as allometric growth laws or exponential functions.

In the equation

$$\phi = (0.6)2^{-z}$$

a = ?  a=0.6  

b = ?  b=2  

c = ?  c= -1

The constant b is referred to as the base.

Recall that in relationships like $y=10^a$, a is the power to which the base 10 is raised in order to get y.
The porosity-depth relationship is often stated using a base different than 2. The base which is most often used is the natural base e and e equals 2.71828..

In the geologic literature you will often see the porosity depth relationship written as

$$\phi = \phi_0 e^{-cz}$$

$\phi_0$ is the initial porosity, c is a compaction factor and z - the depth.

Sometimes you will see such exponential functions written as

$$\phi = \phi_0 \exp^{-cz}$$

In both cases, e=exp=2.71828

Waltham writes the porosity-depth relationship as

$$\phi = \phi_0 e^{\frac{z}{\lambda}}$$

Note that since z has units of kilometers (km) that c must have units of km$^{-1}$ and $\lambda$ must have units of km.

Note that in the above form $\phi = \phi_0 e^{\frac{z}{\lambda}}$ when $z=\lambda$,

$$\phi = \phi_0 e^{\frac{\lambda}{\lambda}} = \phi_0 e^{-1} = 0.368 \phi_0$$

$\lambda$ represents the depth at which the porosity drops to 1/e or 0.368 of its initial value.

In the form $\phi = \phi_0 e^{-cz}$ c is the reciprocal of that depth.
Logarithms

Above, when we talked about functions like
\[ y = ab^x \] and
\[ y = a10^x \]

b and 10 are what we refer to as bases. These are constants and we can define any other number in terms of these constants raised to a certain power.

**Given any number y, we can express y as 10 raised to some power x**

\[ i.e. \; y = 10^x \]

Thus, given y = 100, we know that x must be equal to 2.

\[ y = 10^x \]

**By definition, we also say that x is the log of y, and can write**

\[ \log y = \log \left(10^x\right) = x \]

So the powers of the base are logs. “log” can be thought of as an operator like x and ÷ which yields a certain result. Unless otherwise noted, the operator “log” is assumed to represent log base 10. So when asked what is

\[ \log y, \; where \; y = 45 \]

We assume that we are asking for x such that

\[ 10^x = 45 \]
Sometimes you will see specific reference to the base and the question is written as
\[ \log_{10} y, \text{ where } y = 45 \]

\[ \log_{10} y \] leaves no room for doubt that we are specifically interested in the log for a base of 10.

One of the confusing things about logarithms is the word itself. What does it mean? You might read \( \log_{10} y \) to say - "What is the power that 10 must be raised to to get y?"

**How about this operator?** -
\[
\text{pow}_{10} \rightarrow y
\]

The power of base \( 10 \) that yields \( \rightarrow y \)

\[ \log_{10} y = 1.653 \]

---

**Earthquake Seismology Application**

**The Gutenberg-Richter Relation**

What do you think? Are small earthquakes much more common than large ones?

Fortunately, the answer to this question is yes, but is there a relationship between the size of an earthquake and the number of such earthquakes?
When in doubt - collect data.

<table>
<thead>
<tr>
<th>m</th>
<th>N/year</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.25</td>
<td>537.03</td>
</tr>
<tr>
<td>5.46</td>
<td>389.04</td>
</tr>
<tr>
<td>5.7</td>
<td>218.77</td>
</tr>
<tr>
<td>5.91</td>
<td>134.89</td>
</tr>
<tr>
<td>6.1</td>
<td>91.20</td>
</tr>
<tr>
<td>6.39</td>
<td>46.77</td>
</tr>
<tr>
<td>6.6</td>
<td>25.70</td>
</tr>
<tr>
<td>6.79</td>
<td>16.21</td>
</tr>
<tr>
<td>7.07</td>
<td>8.12</td>
</tr>
<tr>
<td>7.26</td>
<td>4.67</td>
</tr>
<tr>
<td>7.47</td>
<td>2.63</td>
</tr>
<tr>
<td>7.7</td>
<td>1.81</td>
</tr>
<tr>
<td>7.92</td>
<td>0.66</td>
</tr>
<tr>
<td>7.25</td>
<td>2.08</td>
</tr>
<tr>
<td>7.48</td>
<td>1.65</td>
</tr>
<tr>
<td>7.7</td>
<td>1.09</td>
</tr>
<tr>
<td>8.11</td>
<td>0.39</td>
</tr>
<tr>
<td>8.38</td>
<td>0.23</td>
</tr>
<tr>
<td>8.59</td>
<td>0.15</td>
</tr>
<tr>
<td>8.79</td>
<td>0.12</td>
</tr>
<tr>
<td>9.07</td>
<td>0.08</td>
</tr>
<tr>
<td>9.27</td>
<td>0.04</td>
</tr>
<tr>
<td>9.47</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Observational data for earthquake magnitude (m) and frequency (N, number of earthquakes per year with magnitude greater than m)

What would this plot look like if we plotted the log of N versus m?

This looks like a linear relationship. Recall the formula for a straight line?
The Gutenberg-Richter Relation

\[ \log N = -bm + c \]

- \(b\) is the slope
- \(c\) is the intercept
In passing, also note that the magnitude axis can be replaced with the square root of fault plane area since earthquake magnitude is proportional to the square root of area of the fault plane along which rupture occurred. This relationship is linear in the log-log plot shown above right.

Based on the preceding comment, it should be no surprise that we also find a similar relationship between the number of faults of length greater than or equal to a given size at a particular outcrop location.

Another nice attribute of logs is that when you plot the log of y (i.e. the power that 10 (or some other base) has to be raised to to yield y) rather than y itself, it is easier to see relative differences in the value y.
For example, replotting the $N$ vs $L$ data on logN vs. log(L) scale we get a straight line and can easily see the relationship of differences in $N$ relative to size $L$.

In this case, we see that the log-log relationship is linear.

Sometimes the labels on the axes will consist only of the exponent (log or power) itself. Thus, as shown below, the gridlines are labeled $10^0$ or just 0, $10^1$ or just 1, etc.
The Gutenberg-Richter Relation

\[ \log N = -bm + c \]

is linear in a log-log format because \( m \) is earthquake magnitude and you have heard that an earthquake magnitude of 5, for example, represents ground motion whose amplitude is 10 times that associated with a magnitude 4 earthquake.

One of the most commonly used “Richter magnitude” scales determines the magnitude of shallow earthquakes from surface waves according to the following equation

\[ m = \log_{10} \frac{A}{T} + 1.66 \log \Delta + 3.3 \]

where \( T \) is the period in seconds, \( A \) the maximum amplitude of ground motion in \( \mu m \) (10^{-6} meters) and \( \Delta \) is the epicentral distance in degrees between the earthquake and the observation point.
We’ve already worked with three bases - 2, 10 and e. Whatever the base, the logging operation is the same.

\( \log_5 10 \) asks what is the power that 5 must be raised to to get 10.

\[ \log_5 10 = ? \]

**How do we find these powers?**

\[ \log_5 10 = \frac{\log_{10} 10}{\log_{10} 5} \]

\[ \log_5 10 = \frac{1}{0.699} = 1.431 \]

*thus* \( 5^{1.431} = 10 \)

In general,

\[ \log_{\text{base}} (\text{some number}) = \frac{\log_{10} (\text{number})}{\log_{10} \text{base}} \]

or

\[ \log_b a = \frac{\log_{10} (a)}{\log_{10} b} \]

**Try the following on your own**

\[ \log_3 7 = \frac{\log_{10} (7)}{\log_{10} 3} = ? \]

\[ \log_8 8 \]

\[ \log_7 21 \]

\[ \log_4 7 \]
You will find that $\log_{10}$ is often written as $\log$, with no subscript. 

**$\log_{10}$ is referred to as the common logarithm**

$\log_e$ is often written as $\ln$.

**thus**

$\log_e 8 = \ln 8 = 2.079$

$\log_e$ or $\ln$ is referred to as the natural logarithm. All other bases are usually specified by a subscript on the log, e.g.

$log_5$ or $\log_2$, etc.

---

**For Next Time**

*Finish reading Chapters 1 and 2 (pages 1 through 38) of Waltham*

*After we finish our basic review, we will learn how to use Excel, a scientific computing and graphing software package, to solve some problems related to the material covered in Chapters 1 and 2.*