\[
dh \over dt = k \frac{d^2 h}{dx^2}
\]

\[
d(ocean) = \frac{\rho_a}{\rho_a - \rho_w} \left[ 2\alpha(T_w - T_a) \sqrt{\frac{kt}{\pi}} \right]
\]

\[
\phi = \phi_0 e^{-z/\lambda}
\]

\[
\ln(\phi) = \ln(\phi) - z/\lambda
\]

\[
T = 2.303Q(\log_{10} t_2 - \log_{10} t_1)/4\pi(s_2 - s_1)
\]
Geology 351: Geomathematics Lab Manual

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Geology 351-Geomathematics
Syllabus

T-Th 10:00 – 11:15 am

The class web site is located at http://www.geo.wvu.edu/~wilson/geo351.htm
Keep up-to-date on the academic schedule at http://provost.wvu.edu/academic_calendar

Prerequisites - Geology 351 is offered only to geology majors as an alternative to Math 156 and does not fulfill an elective requirement for the minor. Geol 351 builds on your prior experience with algebra, trigonometry and calculus. Geology 351, for the most part, employs math skills you already have and introduces new concepts that will be helpful in future geology classes, as well as calculus or other math classes.

Geology 351 provides a review of math basics and use of spreadsheets to develop problem solving, data manipulation, and plotting skills useful in other upper division geology classes. Geol 351 provides several examples of math applications in geology.

The course provides a review of mathematical methods in geology with an emphasis on geologic examples. You will find that Waltham’s text Mathematics: A simple tool for geologists will serve as a useful present and future reference text. Problems taken from Waltham will give you the opportunity to refine and sharpen your mathematical and EXCEL skills.

Be prepared to discuss and work through problems together but clearly outline and explain your individual work and approach. This ensures you will be successful on exams.

Stay up-to-date! Check the academic calendar at
http://provost.wvu.edu/academic_calendar and final exam schedule.

TEXTBOOK AND LAB MANUAL FOR THE CLASS

- Mathematics: A simple tool for geologists by David Waltham, Blackwell Science, Inc.(Required) This text provides good summaries and applications of logarithms, quadratic equations, exponentials, trigonometry, differential and integral calculus. Contains worked examples. If you work as a geologist in the future you will find this
text a valuable reference. Waltham's text also incorporates use of EXCEL into the analysis of problems and you will find downloadable EXCEL files on his web site. A lab manual is also provided with the course. You can access it at http://www.geo.wvu.edu/~wilson/geomath/labman.pdf or look at individual lab exercises in the table of linked lecture topics below.

**Office Hours:** Generally available for questions following the Tuesday and Thursday classes from 11:15 to noon. Check via email to arrange alternative meeting times.

- **Grading:** Your grade is based on homework, in-class assignments, in-class group activities and exams. Homework, attendance, in-class assignments and computer lab assignments count for 60% of your final grade.

It is essential that you attend class and keep current with reading in the text. All assignments, laboratory exercises, etc. are due at the start of class.

**Late Homework Policy:** There is a 10% deduction for homework not handed in when due. An additional 10% will be deducted for each additional day late thereafter. Homework assignments will not be accepted once that assignment has been returned and discussed. I usually have homework back the following class period. This rule will be strictly enforced unless you provide an official university excuse. If you miss class, you can complete inclass worksheets and receive up to 50% credit if completed and submitted the following class period.

The midterm Exam counts 20% and final in-class activities also count 20%. While the final requires skills developed over the semester, the main focus of the final will be on materials covered in the last half of the semester.

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**Semester outline by topic** – Also see the class web page at www.geo.wvu.edu/~wilson or directly using the link at the top of the syllabus.

**Stay up-to-date!** Check the academic calendar at http://provost.wvu.edu/academic_calendar and final exam schedule.

The list of topics below serves as a general reference. First day of class in Geomath will be on Tuesday, January 10th. Spring Recess extends from Saturday, March 4 thru Sunday, March 12. Midterm exam is scheduled for Thursday, February 23rd in rm 325 Brooks. Final in-class activities April 25th and 27th (stay tuned for details).

For daily lecture content see table with links below

Lectures and other content are updated daily and good primarily as a current reference. Youtube videos linked below review some of the topics discussed in class.

For more exciting tutorials visit your Mathbff and Kahn Academy.

Having trouble solving that equation - visit WolframAlpha.

<table>
<thead>
<tr>
<th>general content</th>
<th>Reading/Problems Assignment</th>
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<tr>
<td>1. (Jan 10th) First Day Activities: Some problems concerning sea level rise, water use, and shrinking ice caps that illustrate contemporary quantitative applications in geology. See In-class problem set related to 2 of Waltham and the in-class</td>
<td>Order your book ASAP! Begin looking through chapters 1 and 2 of Waltham and the in-class</td>
</tr>
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intro materials (Chapters 1 & 2). Order now! For more about the text and author visit [Waltham's page](#).

<table>
<thead>
<tr>
<th>Date</th>
<th>Topic</th>
<th>Assignments</th>
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<tbody>
<tr>
<td>2. (Jan 12)</td>
<td><strong>Introduction I</strong>: Elementary mathematical relationships in geology. <strong>Group Problems;</strong> Units conversion examples and answer key (click here)</td>
<td>Chapters 1 and 2 of Waltham (pages 1-38). Video discussion of subscripts, exponential notation, and linear relationships.</td>
</tr>
<tr>
<td>3. (Jan 17)</td>
<td><strong>Introduction II</strong>: Expressing relationships between geological variables. Today's group problems for review.</td>
<td>Chapters 1 and 2 of Waltham (pages 1-38). For a reminder on the assumptions made in these simple equations see <a href="#">youtube</a> video.</td>
</tr>
<tr>
<td>4. (Jan 19)</td>
<td><strong>Math Relationships</strong> continued. Have a look at basics.xls (for a quixk tour of the basics.xlsm file, see this video) and additional problems for group discussion today. Another in-class problem.</td>
<td>Discussion of Archie's law part 1 and 2. The physical relationship of F to porosity. Video discussion of why F decreases as the porosity increases.</td>
</tr>
<tr>
<td>5. (Jan 24)</td>
<td>Using Excel to solve problems 2.11 and 2.12: slides and Computer Lab 1: Problem Presentation Format &amp; related concepts associated with the North Sea problem. For a more detailed view of the North Sea data analysis see videos part 1 and part 2.</td>
<td>Chapter 2 problems 2.11 and 2.12 Excel analysis of north sea data from Video related to part iii) in the computer lab: calculating the intercept.</td>
</tr>
<tr>
<td>6. (Jan 26)</td>
<td><strong>Today's discussion</strong> reviews mathematical models covered to date, introduces the Fourier series and introduces problem 2.13. See Computer Lab 2 (will be handed out in class) which will cover use of Excel to solve problem 2.13 and general radioactivity decay problems. See Step2.xls and have some fun with Fourier Series. Today's in-class activity.</td>
<td>Chapter 2 problem 2.13; add 2.15 to the homework assignment. YouTube video review of the in-class activity.</td>
</tr>
<tr>
<td>7. (Jan 31)</td>
<td>Discussion of radioactive decay problem 2.13 continued with general discussion slides; Some in-class problems associated with exponential functions.</td>
<td>Video discussing logarithmic transformations of exponential functions: Parts 1 and 2. This <a href="#">youtube video</a> guides you through the in-class activity.</td>
</tr>
<tr>
<td>8. (Feb 2)</td>
<td><strong>For the day</strong>: Review exponential decay and growth relationships, their solution, intercept determination, organization of presentation and basic equation manipulation. This Excel file provides several worksheets that illustrate the population problem and series approximation of the base e (see Ln_Expansion.xlsx). An in-class problem related to isostatic equation manipulation; and consider problems 3.10 and 3.11 for group discussion next time. Visit this useful site for units conversion.</td>
<td>Read Chapter 3: look over problems 3.10 &amp; 3.11. Video presents some background on the development of Stoke's law.</td>
</tr>
<tr>
<td>9. (Feb 7)</td>
<td><strong>Isostasy</strong>: The basics of equation manipulation illustrated using problems in isostacy. Try this in-class problem. Look over this additional problem take-home isostacy problem concerning isostatic adjustments during long term erosion. We will discuss in class next time and turn in. Questions about problems 3.10 and 3.11 worksheet handed out last time? Turn in my mailbox This Thursday by noon.</td>
<td>Continue your reading Chapters 3 &amp; 4</td>
</tr>
</tbody>
</table>
10. (Feb 9) More on **Isostacy**, then **Computer Lab: Roots of quadratics and operator preference** issues (for reference only); see also Quad.xls. We'll spend most of our time on Stokes Law and the settling velocity problem. Turn in book problems 3.10 & 3.11. Readings: Chapters 3 and 4.

- Settling velocity Excel problems due Tuesday 23rd.
- Problems 4.7 and 4.10 due March 1st
- Fitting lab due March 3rd.

<table>
<thead>
<tr>
<th>Getting close to midterm</th>
<th>Test coming up ... start preparing</th>
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<tr>
<td></td>
<td>Start reading chapter 8</td>
</tr>
<tr>
<td>12. (Feb16) Catch up if needed</td>
<td>Pre-Exam review ... review</td>
</tr>
<tr>
<td>13. (Feb21) Pre test review (slides): Practice Exam Questions</td>
<td></td>
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</tbody>
</table>

14. Mid Term Exam (February: Thursday 23rd, rm 325 Brooks Hall) | Mid Term Exam held in rm 325 |

15. (Dates approximate ~ Feb 28) Post-test discussion. Get started reading Chapter 8

16. (Mar 2) **Slides for the day.** Complete the fitting computer lab. Hand problems 4.7 and 4.10 before leaving.

**Spring Break (Mar4 through 12)**

17. (Mar 14) Some **initial thoughts** on derivatives. Have a look at these **problems for discussion** and consider the slopes of common functions. Start working through these example derivative **problems**. Turn in fitting lab exercise. Continue reading Chapter 8.

18. (Mar 16) Derivatives continued - **Lecture Overheads (continued):** Visualize your formula at WolframAlpha & also see See Excel examples in limits.xls. Finish in-class basic differentiation **problems** and basic differentiation rules. Some self-assessment questions. Finish reading of Chapter 8


20. (Mar. 23) Visiting lecturer with assignment (don't miss download related excel file at GeomathFun). **Slides for today. Discussion of text problems 8.13 and 8.14.** Computer problems 8.13 and 8.14 get started and bring questions to class Tuesday following Spring Break. Questions about the self review problems (handed out last class). Text problems 8.13 and 8.14 are due this Thursday.

22. (March 30st) General slides. Introductory discussions of integral calculus (the antiderivative or derivative in reverse). Derivatives in reverse - group exercises 1 and 2. Extra credit exercise using WolframAlpha.

23. (April 4) Introduction to Integral Calculus: Definite and indefinite integrals. Calculating lithostatic and hydrostatic pressure at depth. Will spend time in class in groups but not due till next time. Some simple calculations using Waltham's integ.xls file. See also area.xls for an analysis of error between the discrete approximation and exact integrations. In class integral assessment problems - hand in before leaving.


25. (April 11) Integral Calculus (continued): Return to the group integration problems from last time and discuss. Questions on the fold shortening problem. Consider the volume of Mount Fuji and bar sand volume problem Question 9.7 to be handed out.


27. (April 18) Slides for the day. Questions on problems 9.9 and 9.10. Spreading ridge mass problem and another final review problem.

28. (April 20) Slides for the day. Excel files referred to in slides are on the common drive. Questions about the review problems from last time? Review problem 2.

Preparation for the final through in-class group efforts


30. (April 27): Brief slides for the day. 2nd day of in-class/take-home activities. Turn in data analysis activity before leaving. Reference slides and review problems from last week may be of interest.

Exam schedule - 5-7pm, Thursday, May 4th

Grade summary. Begin Exam review.
Internet resources for Waltham’s text *Mathematics for Geologists*

Bring up your internet browser (Netscape or Microsoft Explorer) and enter the following address http://davidwaltham.com/mathematics-simple-tool-geologists/ into the URL location list box. The following screen will appear –

Click the *Maths for Geologists* link for additional information along with links to Excel files accompanying the text. These will be provided on the common drive.

<table>
<thead>
<tr>
<th>SPREADSHEET</th>
<th>SECTION IN THE BOOK</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intro.xls</td>
<td>1.5 &amp; 1.8</td>
<td>Introductory sheet showing scientific notation and graphing.</td>
</tr>
<tr>
<td>S_line.xls</td>
<td>2.2</td>
<td>Plots straight line given m &amp; c.</td>
</tr>
<tr>
<td>Quadrot.xls</td>
<td>2.3</td>
<td>Plots quadratic function given a, b &amp; c.</td>
</tr>
</tbody>
</table>
If you click on one of the links (Quadrat.xls for example) you’ll get the following prompt.

Open to see contents:

We’ll spend a little time going over all this in class, and there will be plenty of opportunities to gain experience working with Excel. Remember that one of our
main objectives for the semester is to develop some basic computer problem solving skills that will be of use to you on the job as well as in other Geology courses.

Notes, Notes … for Future Reference
Evaluating Depth/Age Relationships in the North Sea
The following instructions take you step-by-step through the generation and plotting of a data set using Excel. Many of you may already be familiar with Excel, but if you aren’t that’s not a problem. In future exercises, you will learn how to fit straight lines and polynomials of higher order to specific data sets. Today, we are going to plot up some data and take a conceptual look at some linear relationships we will see within the context of age-depth relationships for sediments deposited in the North Sea. We will learn how to generate xy data plots and format them in various ways.

GETTING INTO EXCEL
Follow along as we bring up Excel. Access to various controls are arranged around the “ribbon” across the top of the Excel window. There are several folders across the top that one accesses to perform various tasks. To open a file, go to the office button in the upper left corner and click on it. In the drop down list you will see an open file option. Left click on FILE then browse to your data folder. In most cases this might be on the Desktop under your favorites list (upper left). A lot of the class files will reside on a network folder. Before getting into Excel go to the “common drive” and copy the FittingLabData folder over to your N:\Drive. The N: Drive is another network drive that is linked to your account and can be accessed from any computer in the building that is on the network. This is your personal file storage area. We’ll spend some time in class reviewing all this (take notes!). For today’s lab, select the data set DepthAge.xls and click the open button. A spreadsheet or window containing the depth and age data will appear on the screen.

The data in the spreadsheet were taken from Chapter 2 of Waltham’s text (page 37, problem 2.11). The following data shown in the display below were taken from the Troll 3.1 well in the Norwegian North Sea.


The first row contains the column labels Depth and Age (in cells A1 and B1) defining the data type listed in each column.
We’ll spend a little time setting up some Quick Access tools so you know they are there and can hunt down additional ones as needed.

Click on the double down arrows above and start pushing commonly used functions onto the quick access bar.
GRAPHING AND PLOTTING!

Let’s see what this data looks like.

*Click the insert folder tab on the Ribbon (note variety of options)*

*Click on the Scatter plot icon in your Quick Access toolbar.* You’ll see a variety of plot styles in the drop-down window. Select plot style with data points and curved line fit.

After you click on the Scatter plot option a blank plot window comes, select columns A and then B to specify the x and y plot data (follow along in class and take marginal notes).

Follow along and **MAKE NOTES** as we go through these examples.
Now that you’ve made this nice plot delete it!

**NOTES**

Another way – First select the data and then select the scatter plot option again. The plot should automatically come up (see next page). Let’s put this plot option in your Quick Access Toolbar. Right click on the scatter plot option and select Add to Quick Access Toolbar.

**NOTES! NOTES!**

Plot generated by selecting the columns first. This plot also has formatted axis labels and chart layout.

**ODDS and ENDS -**
**Axis Labels**
Click on axis labels and correct or change as needed. Add units in parentheses: eg Depth (cm) & Age (years).
**General Formatting**

From Format > Quick layouts > select the style of plot you’d like
Also right click on the plot line and select **Format Data Series**.

Experiment with the Marker Fill, Marker Line Color selection windows
Review:
How do you bring in your data?

How do you plot your data?

How can you change the plot line style (color, thickness, continuous or dashed…)?

How can you change the data point display style (marker line style, line color, …)?

How do you get back to the Chart Tools?

How do you change the chart layout?

How do you format gridlines, chart axes?

How can add a chart title?

How do you move the graph around in the chart window?

What are the three tabs available under Chart Tools?

Do you have any questions?
Printing your plot
Send your plot to the printer, but first change the title of your plot to include your name.

If you don’t have a title, go to go to the Add Chart Elements icon at left. Check out the More Title Options. Or, if you already have a title on your chart, right click and select Format Chart Title.

You can change axis labels the same way.

You can also add a text box. A good item to put in your Quick Access toolbar. The text box along with the elements in your Shapes drop down will allow you to label, add arrows and point out features of interest.

Try this out now.

Printouts go to the printer room between the labs

NOTES:
North Sea Depth-Age Data (your initials)

Click on your text box and arrows; move them around. Change their formats.

Slope of the deeper sediments is less than that for the shallower, younger, sediments. What does this mean?
**Back to Problem 2.11:** From the graph and data listing, estimate the sedimentation rate for the last 10,000 years, (ii) the sedimentation rate for the preceding 5,000 years, and (iii) the time since sedimentation ceased.

![Graph showing sedimentation data](image)

i) Compute the sedimentation rate \( \frac{\Delta \text{Depth}}{\Delta \text{Age}} \) from data points using depths extending from 19.75 cm to 407 cm and ages of 1,490 years to 10,510 years. This yields: \( \frac{387.25}{9020} = 0.043 \text{ cm/year} \) (or 23.29 \( \text{ years/cm} \)).

ii) This question can be answered in a couple ways: we could fit a regression line to the data with ages of 10,500 years and greater; or we could make a quick estimate by using the end points on the line and assume the age varies linearly along a line between those two points. So we just compute the rate the same way we did in part i) but using the end points shown above.

iii) To answer this question we have to find the slope of the line considered in part i. Using the equation of a line \( \text{Age} = \text{Slope} \times \text{Depth} + \text{Intercept} \) you then solve for the intercept. Remember you know that the depth equals zero at the surface so the intercept is the age of the sediments at the surface. What else have we assumed in making this estimate?
GENERATING A LOG PLOT -

Remember from our in-class discussions of possible age depth relationships we suggested increased depth of burial would compact sediments and reduce their porosity. Thus the age-depth relationship could be variable through time. Increased depth of burial would increase the amount of time represented by a unit thickness. For example, a meter thick section of strata buried at 5000 meters might have taken 40,000 years to deposit, whereas a meter thick section in the near-surface might span only 10,000 years or so. Could the age of these North Sea sediments increase exponentially with depth in the form $A = A_0e^{ad}$?

Do you remember how to take the natural logarithm of both sides of this equation? What kind of equation results from this operation?

Let's check it out.

Go to **Chart Tools >> Layout >> Axis >> Primary Vertical Axis >> More Primary Axis Options** and check the logarithmic scale check box. You also have the option to change the min and max values along with other parameters. Note in the figure below that the Age axis and gridlines have been rescaled to equally proportioned logarithmic intervals (base 10).

Does your plot look more like a straight line?

Here's some data where age does increase exponentially with depth.
Now that you’ve gone through the basics of constructing a graph or chart and printing it off, take some
time to go back over what you’ve done and play around with the different formatting options. The best
way to learn will be to experiment.
Save your data. The best place to save will be on your \texttt{N:drive}.

Go to your Office Button (Upper left)

Click on the Save or Save as icon and use the \texttt{N:drive}

Give your file a name you will remember like problem 2-11.

Files saved on the \texttt{g:drive} can be accessed from other computers in this lab or other Brooks Hall computers tied into the department computer network. If you save to the \texttt{c:drive} on the machine you are currently using you will only be able to access your data from that machine.
**Problem 2.12 (computing liquid phase concentrations)**

As crystals settle out of magmas, the element concentrations (C in formula below) in the remaining liquid fraction change according to the equation

\[ C = C_0 F^{(D-1)} \]

where \( C_0 \) is the initial concentration of the element in the liquid before crystallization began, \( F \) is the fraction of liquid remaining and \( D \) is a constant (known as the distribution coefficient). Calculate the concentration of an element after 50% crystallization (i.e. \( F = 0.5 \)) if its initial concentration was 200ppm and \( D = 6.5 \).

Let’s take a different approach to the solution of Problem 2.12. Rather than solving \( C \) for just one value of \( F \) let’s solve \( C \) for a range of \( F \)s extending from 0 to 1 at intervals of 0.05. This way, we get a comprehensive look at the function and how it varies.

This will give us a total of 21 computations of \( C \). Sounds like a lot of work, but we’ll use some autofill operations and can probably do all this in the time it would take you to do one computation by hand.

**First -**

Move into the second spreadsheet in your Excel Workbook. While we’re at it, let’s give each spreadsheet a name. Call Spreadsheet 1 Prob2-11 and Spreadsheet 2 Prob2-12. Just double click on the sheet tabs and you’ll find that you can type names and format these Sheet tab labels.

Next identify your variables in row 1 cells A1 and B1. Type \( F \) into cell A1 to indicate that column 1 will contain values of remaining liquid fraction. Type \( C \) into cell B1 to indicate that column 2 contains the calculated element concentrations. Since \( F \) is our independent variable, we need to enter values of \( F \) over the range from 0 to 1 in increments of 0.05. To do this type in the first two values: 0 in Cell A2 and 0.05 in Cell A3.

Then left click on Cell A2 and drag down to Cell A3 and lift up on the left mouse button. Your screen should look like that shown at left.
To fill out the remaining cells from 0.1 to 1 just right click on the lower right corner of the highlighted cells and drag down the column. Note that as you drag down the column, there will be a little post box next to the current location of the mouse arrow telling you what the value of F is in that cell so you know when to stop! Your sheet should look like that shown below.

Now that you have your independent variables specified over the range 0 to 1, we are ready to calculate the values of $C = C_0 F^{(D-1)}$. Before we do this though add a third column title CO in cell C1 and then enter the value 200 in cell C2 (see figure above). Then highlight cell C2 and assign a variable name CO to this cell in the cell identification box above Cells A1 and A2 (see circled entry below).
Now we are ready to calculate element concentrations C. In cell B2 enter the formula

\[ C = C_0 F^{(D-1)} \]

* represents the multiplication operator

^ represents the exponentiation or power operator

CO is the initial concentration of 200 ppm
D is the distribution coefficient with value 6.5

The equation says

\[ C = 200 F^{(6.5-1)} \]

Now, highlight this single calculation cell (Cell B2) and then place your mouse on the little black handle on the lower right of the cell box. When you get the plus sign, left click and drag down to cell 22. Note all values of C have been automatically calculated (see below).
How does concentration (C) vary with liquid fraction (F)? Remember how to generate a plot? Select columns A and B; Click on the Ribbon Insert tab; Select the scatter plot; under the Chart Layout options drop down list in the design tab, select the gridded style plot (lower left). Under the Layout Tab specify labels, titles, etc. Put those professional touches on your plot display.

In the above setup we specifically assigned values to the variable C₀ and D. While you have your plot up change these values and note that the values of C are automatically recalculated and your plot is updated.
Bring questions to next class. We’ve covered a lot of ground today so make sure you spend some additional time outside of class working over the procedures we’ve covered.

Due date for Problems 2.11 and 2.12 to be decided-

Presentation Outline – Computer Problem Set 1
Problems 2-11 & 2-12

2-11
a) Present the graph of Age versus Depth (5 points).

Label to note regions with different sedimentation rates.

b) Present your calculations. Organize them in a step by step fashion. Don’t just write down the answer. Show the details.
   i) What was the sedimentation rate during the past 10,000 years? (2 points)
   ii) What was the sedimentation rate during the preceding 5000 years (~ 10,000 to 15,000 years ago)? (4 points)
   iii) When did sedimentation cease? (4 points)

2-12
a) Present your graph. (5 points)
b) Present hand calculations of the concentration after 50% crystallization. (3 points)
c) Using your calculated data tables compare the change in concentration from 85% to 75% liquid fraction with that occurring between 55% and 45%. (2 points).
Basic Radioactive Decay/Age Relationships
You received a pretty thorough introduction to Excel in the solution in the lab guide along with the work you are doing to solve problems 2.11 and 2.12. Some of this may be a little redundant but will provide some basic review and opportunities to ask additional questions about Excel functionality.

GETTING INTO EXCEL
Bring up Excel. You should have a view of an empty spreadsheet as before.

- Consider problem 2.13 from Waltham’s text.

Problem 2.13: Radioactive minerals become less radioactive with time according to the equation

\[ \ln(a) = \ln(a_0) - \lambda t \]

Where \( a \) is the radioactivity (in counts per second), \( a_0 \) the initial radioactivity, \( t \) is the time and \( \lambda \) is a constant which depends upon the mineral. If \( a_0 = 1000 \) counts per second and \( \lambda = 10^{-7} \) y\(^{-1}\), draw up a table and plot a graph of \( \ln(a) \) against \( t \) for times ranging from 0 to 100My. From your graph, estimate the age of a specimen which has decayed to \( a = 100 \) counts per second.

Spreadsheet Operations
The direct computation of \( t \) in the above equation is fairly trivial algebra problem. As you can see from the equation above it is linear and we only need two points, so estimating the age could be done graphically without resorting to the computer. However, we’ll expand the nature of the problem to include analysis and plotting of the \( \ln(a) \) relationship and also the radioactivity equation in its exponential form. We’ll make our computations of \( \ln(a) \) from times \( t = 1 \) to 100 MYA at intervals of 1 MYA - i.e. for a total of 100 computations.

As usual **TAKE NOTES** as we go through these procedures!
As we did the last time, define variables $A_0$ and $\lambda$ (see figure below). We’ve also inserted a value check there for the computation of $\ln(100) = \ln(100)$.

Type in your value for $A_0$ of 1000, then in the upper left name box enter $a_0$. Insert value for $\lambda$ of $1e7$ and name it $\text{lam}$.

Remember that EXCEL will auto-fill values of the independent variable from 1 to 100. All we need to do is give it the first two numbers in the series.

The procedures for doing this are outlined in the window at right.

- **Label your data columns Time and ln(a) (see figure below)**
- **Type in the first number 0, then in cell A2, 1e6 (in cell a3). Note that 1e6 stands for $10^6$ or $1000000$.**
- **Type in the second number in the series, 2e6, in cell a3.**
- **Select those two cells (A3 and A4)**
- **Place the mouse arrow on the lower right corner of cell A4, click and hold left mouse button down and drag down to cell 102. The value in cell A102 should equal 1.00E+08.**
- **Lift up**

Numbers 1 through 100 will appear in cells 3 through 102.

Click, hold, & drag; the mouse tip shows the value that will be dropped in the current row.
Enter computations of \( \ln(a) = \ln(a_0) - \lambda t \) in column B.

EXCEL computations make reference to specific cells in the spreadsheet through their row-column identifier. Cell 1 in column A is referred to as A1, cell 2 as A2, etc.

EXCEL directly translates mathematical expressions preceded by an = sign into numeric results if all constants are specified. For example. Type =ln(100) into the value check cell shown below (cell E2). Enter and the value 4.60517 should appear.

\[
\ln is your natural log and always implies base e. In the case of the Gutenberg Richter relationship we are using base 10.

Remember logN implies \( \log_{10}N \); \( \ln N \) implies \log_{e}N. Make sure you don’t use \( \ln \) when you should be using log base 10!

The radioactive decay equation we are working with is derived from \( a = a_0e^{-\lambda t} \). To get the equation you are working with in this problem you would take the natural log (ln).

In the presentation that follows, we’ll also show you how to use the constants without giving them names.

Naming your constants can really help make complicated computations easy, but are not always necessary!
Given that $A_0 = 1000$, recall $\ln(1000)$ (the intercept in this equation) represents the natural log of 1000. It is the power that the natural base $e$ is raised to, to obtain the number 1000. Remember, the result - 6.907 - tells us that $e^{6.907} = 1000$.

$\ln(1000) = \ln(a_0)$ in the equation: $\ln(a) = \ln(a_0) - \lambda t$ or

We don’t have to define variable names as we did above. As an alternative, we can just write them in as constants:

$\ln(a) = \ln(1000) - 10^{-7} t$ - after substitution for the constants $a_0$ and $\lambda (10^{-7})$.

Remember the $t$'s are listed in column A. To calculate $\ln(a)$ return to cell 1 in column B and type in

$=\ln(1000)-(1e-7)*A2$ (i.e., $\ln(a) = \ln(a_0) - \lambda t$) or $=LN(a0)-lam*A2$

$1e-7$ means the same thing as $1 \times 10^{-7}$ and the specific value that we are calculating is the value of $t$ stored in cell A2.

The numeric result appears in B2 (at right). In the function display box (fx), note that the has been changed to 0.0000001.
Now, with cell B2 highlighted (which ever format you’ve opted to use),

- Take your mouse pointer over to the lower right corner of the active cell B2 (look for the plus sign to show up)
- Left click on the corner, hold the mouse button down and
- Drag the mouse vertically down column B to cell 102.
- Lift up on the left mouse button and you should see the calculated values of ln(a) as shown below.
Time to review your plotting skills

\[ \ln(a) = \ln(1000) - 10^{-7} t \]

Select the two columns of data: \textit{Time} and \textit{ln(a)}; then click on the quick access short cut or on the “ribbon” click on the \textit{Insert folder} and generate the scatter plot as before (see lab 1). You should end up with a plot that looks similar to that shown below. Remember you can use the shortcut in \textbf{Quick Start} icon list to generate the scatter plot.

We need to spend a little time reformatting this plot to make it more presentable – easier to read.

Check out the \textbf{CHART TOOLS > DESIGN AND FORMAT} tools. See \textbf{Quick Layout} drop list and the \textbf{Add Chart Element} drop list on the left of the Design ribbon.

Pick a \textbf{Quick Layout} and \textbf{Add Chart Elements} as needed. You may have to activate the Home tab to change the Font Size. Don’t forget you can continue to add \textbf{commonly used buttons to your Quick Access Toolbar across the top}. Just do this by \textbf{right-clicking on the items} you want to add for quick access. If you want to move the font type and font size items for Quick Access, right click on the down arrows to get the move option.

Using the Chart Design >>> Chart Layout options, you can easily reformat your plot to include gridlines, but note that when you do this you loose your title. It's easy to add elements to your \textbf{Quick Layout} favorite.
Click on the vertical axis and right click (RC) to bring up the Format Axis options. Drop down the number selections and change the number format to number with 0 decimal places (see below for example). Obviously you can change this to suit your preferences.

Next click on the Axis Options (above Number) and tell the chart to let the horizontal axis crosses y axis value of -4 (see below).
Work on the X-axis format as well. Change the number format to Number with 0 decimal places. Label at intervals of 25,000,000 years (under axis options) and then under Alignment, angle the labels at 45 degrees.

Your plot should look something similar to that shown below.
Recall the basic question we have been asked to solve: Waltham asks you to graph $\ln(a)$ vs. $t$, and then asks for $t$ at a particular value of $a$.

There are three approaches we could take to find $t$ at this value.
1) Taking a graphical approach, we could compute $\ln(a)$ and find the time $t$ corresponding to that value of $\ln(a)$.

What is $\ln(100)$? Remember it's easy to compute the result $\ln(100)$ using EXCEL. Just go to a blank cell in the spreadsheet and type in $=\ln(100)$ and then enter. **You should get 4.60517.** Now go to the EXCEL plot window and look at the plotted values. Locate 4.605 on the y-axis and find the corresponding value of $t$.

Well, it's a bit difficult to do very precisely. But Excel mouse tips will reveal an approximate answer. Click on the plotted values of $\ln(a)$ vs. $t$ in your chart. Now take your mouse arrow and move it onto the line. Notice that individual x and y values appear in a rectangular box below the pointer (see below). Now we easily see that the value $t$ corresponding to the value of $\ln(a)$=4.61 (about as close as you can get) is 23 million years.
**Approach 2)** The easiest thing to do would be to solve the equation directly for \( t \).

\[
\ln(a) = \ln(1000) - 10^{-7} t
\]

Subtract \( \ln(1000) \) from both sides

\[
\ln(a) - \ln(1000) = -10^{-7} t
\]

Divide through by \(-10^{-7}\)

\[
\frac{\ln(a) - \ln(1000)}{-10^{-7}} = t
\]

Since \( \frac{1}{-10^{-7}} = -10^7 \),

\[
\frac{\ln(a) - \ln(1000)}{-10^{-7}} = t \quad \text{becomes} \quad t = (\ln(1000) - \ln(a))\times 10^7 .
\]

Now go to a nearby vacant cell in your EXCEL spreadsheet and type in 

\[
=(\ln(1000) - \ln(100))\times 10^7
\]

and the answer 23025851 will be returned.

Waltham asked us to plot the line, which we did; but this wasn’t really necessary to obtain the numerical result. But we have additional objectives. We want to develop proficiency in using computers to analyze and display a variety of geological data. Also, the plot we’ve obtained highlights the linear nature of the radioactivity relationship in its logarithmic form.

**Approach 3)** Here we take the opportunity to discuss the basic relationship from which the logarithmic expression is derived. Recall from lecture that \( \ln(a) \) is the power that the base \( e \) must be raised to to obtain the value \( a \) (You’ll find additional material in sections 2.8, 2.9, and 4.2 of the text. Review these sections for some additional background). What would happen if we did the following?

\[
e^{\ln(a)}
\]

If we raise \( e \) to the power \( \ln(a) \) - which is the power that \( e \) must be raised to to obtain \( a \) - then \( e^{\ln(a)} = a \).

If we take both sides of the equation \( \ln(a) = \ln(a_0) - \lambda t \) as powers of \( e \), the equivalence is retained and we have

\[
e^{\ln(a)} = e^{(\ln(a_0) - \lambda t)}
\]

Recall that \( e^{(\ln(a_0) - \lambda t)} \) is of the form \( x^{a_0 b} \), which is just \( x^a \). Hence \( e^{(\ln(a_0) - \lambda t)} = e^{\ln(a_0)} e^{-\lambda t} \).

Thus \( e^{\ln(a)} = e^{(\ln(a_0) - \lambda t)} \) reduces to \( a = a_0 e^{-\lambda t} \) or in our specific case to \( a = 1000 e^{-10^{-7} t} \).

This form of the radioactive decay equation provides the third, easiest, and most direct method for solving our problem.

In your EXCEL spreadsheet go to the top of column C add the column title **a in cell C1** and in cell C2 type in the expression:

\[
=1000*exp(-1e-7*A1)
\]

Copy the expression into cells 2 through 100 (see figure below).
Now plot column C as the dependant variable
The difference here is that we want to highlight or select two columns of data that aren’t adjacent to each other. To do this just select column A and then, with the Ctrl key held down, left click on column C to select. Now you’ll have both columns highlighted and you can go through the plotting process as you normally would.
Time to do some formatting: you must be getting good at this by now!

Take some time to explore labeling, changing line plot formats, and other features of EXCEL.

We'll review some of this in class and leave time for questions.
Assignment

At this point we’ve worked through most of the details of the assignment and you should have everything you need to complete the items in the list below.

Check-off list for this assignment:
2 points - Your work should include a statement of the problem including the given values in the problem and the value(s) to be determined.

6 points - Submit an Excel plot of ln(a) vs. t and include your name in the Chart Title

2 points - On your chart locate the point with value ln(100). Extend a line down from the appropriate point on the curve to the t-axis and note the value of t corresponding to ln(100) on the t axis. Label that point noting the corresponding value of t. Also, extend a line parallel to the time axis out to the ln(a) axis from your curve locating the value of ln(100) and label that point. You can draw the line in pencil if you wish.

4 points - Show the numerical details of the computation of t (ln(100)), i.e.

6 points - Include an Excel plot of \( a = a_0 e^{-\lambda t} \)

2 points - On your chart locate the point with value a=100cps. Just as in question 2, note the value of t at which the radioactivity drops to 100 cps. Label the values of a and t on the a and t axes.

4 points - Specifically state the results of your analysis in sentence form.

Total 26 points

Complete today's assignment and hand it in

__________________________
In Chapter 3 of *Mathematics: A simple tool for geologists*, Waltham takes us through a brief review of quadratic equations and their roots. The first exercise below is an optional exercise that allows you to explore the graphical significance of roots. You do not need to hand this in and use EXCEL to help us solve problems at the end of Chapter 3.

**Exercise I (Optional):** Graph the following functions:

A) \[ y = 3x^2 - x - 5 \]
B) \[ y = x^2 + x + 3 \]
C) \[ y = -x^2 + 2x - 1 \]

and determine their roots.

Open Excel and generate a column of numbers corresponding to the \( x \)'s in the above equations. The roots of these equations are present somewhere over the interval \(-2 < x > 2\). So fill Column A with \( x \) values that run from -2 to 2 and use a calculation interval of 0.1 (see illustration at right).

Values of \( x \) running from -2 to 2 at a sample interval of 0.1 will occupy 41 cells (rows) from A2 through A42. The first cell has a value -2 and each consecutive cell will have a value \( x \) incremented by 0.1.
Enter the three equations in columns B, C and D.

For example, take the first equation: \( y = 3x^2 - x - 5 \) Use the proper operator notation. So enter \( y=3*A2^2-A2-5 \), as shown at right.

Remember -
* represents the multiplication operator
^ represents the exponentiation or power operator

Click on the formula box and drag down to row 42 to populate y values for this quadratic (Equation A)

What does this quadratic equation look like?

Remember how to generate a plot?

Start by selecting the columns containing your x and y data (see figure at right). Note the short cut icon to the scatter plot in the Quick Access Bar across the top. Did you add one earlier on? It’s often used and now might be a good time to add it in.
After selecting scatter plot with smooth line and markers, your plot will look like that below. Practice your plot formatting skills.

From the design tab select Quick layouts to get grid lines. Under the Layout tab format your horizontal and vertical axes. Add appropriate labels and titles. Use the Line Style option in your format axis command list to highlight the x-axis (see below).

Remember how to format your plot line and data points? Highlight and right click plot elements. For the highlighted data series the format data series option in the drop list.
Where are the roots of the equation $3x^2-x-5$ located? Remember how the root is computed and what values you are solving for when computing the roots.

Next, generate plots of quadratics B and C.

B) $y=x^2+x+3$

C) $y=-x^2+2x-1$

These plots are part of an in-class activity, so just put your name on them and hand in.

**EXERCISE II (REQUIRED FOR GRADE):**

**Problem 3.11** Stokes' law states that the viscosity at which a spherical particle suspended in a fluid settles is given by

\[
v = \frac{2(\rho_p - \rho_f)gr^2}{9\eta}
\]

where $v$ is the velocity of descent, $\rho_p$ and $\rho_f$ are the densities of particle and fluid respectively, $g$ is the acceleration due to gravity, $r$ is the particle radius and $\eta$ is a property of the fluid known as viscosity. Assuming that grains of different sizes have identical densities, show that the ratio of the settling velocities for two different grain sizes is

\[
\frac{v_1}{v_2} = \left(\frac{r_1}{r_2}\right)^2
\]
where \( v_1 \) and \( v_2 \) are the velocities for grains of radius \( r_1 \) and \( r_2 \) respectively. If a grain of radius 0.1 mm, suspended in a lake takes 10 days to settle to the lake bottom, how long would it take a grain of radius 1 mm.

Viscosity is a measure of the resistance of a liquid to flow. The viscosity of water at room temperature is about 0.01 poises. 1 poise is 1 gram/cm-second. The units of viscosity are also often given in pressure-seconds such as Pascal-Seconds (a pascal is one Newton/meter \(^2\)). A thick oil might have a viscosity of about 1.0 poise.

**Using a viscosity of 0.01 poise (gm/cm-s) for the settling of different size sand grains in a lake.** Let the particle sizes range from 0.001 cm to 0.1 cm (i.e. 0.01 mm to 1 mm) and increase the particle radius by increments of 0.001 cm (0.01 mm) over the range. Use \( g = 980 \text{cm/sec}^2 \), \( \rho_{\text{sand}} = 2.67 \text{ gm/cm}^3 \), \( \rho_{\text{water}} = 1 \text{ gm/cm}^3 \).

Set up variable definition cells in column A. In Column B assign the values to viscosity, acceleration due to gravity, and sand and water densities. **Assign variable names** to them as well (e.g. visc, g, RhoS & RhoW).

In your Excel spreadsheet (give it a name if you like such as SetVel or Stokes as noted below) generate 100 values of \( r \) in column C that range from 0.001 to 0.1 (at intervals of 0.001). Enter the equation for velocity in Column D. Your Excel view should look similar to that shown below. Fill in the velocities for each value of \( r \) in Column C. Following the setup illustrated here, that should give you a value for 0.1 cm in row 102. Remember 0.001 cm is 0.01 mm and 0.1 cm is 1 mm. Enter the formula as shown below. Be sure to put \((2*\text{visc})\) in parentheses.

Select Columns C and D (\( r \) and \( v \)) and **generate a plot**. Double check your result with that shown below. Format the chart axes so that the plot covers relevant data intervals (i.e. 0<\( r <0.1 \)).
You can use the quick layouts to get a plot setup with axis labels, etc.

- Given the specific viscosity for water, we have been able to model settling velocities as a function of particle radius. In the text you were able to answer questions concerning ratios and settling times. You did not have to know the velocities to do this. Using your computed velocities, determine the velocity that a grain with radius 0.1 mm settles in the lake. **If it takes that grain 10 days to settle to the bottom of the lake, how deep is the lake?**

What units should time be in?
What units will the lake depth be in?
Convert lake depth into units of kilometers.

- Next, construct a plot of settling time versus particle size using a lake depth of 100 meters. Remember, the time it takes for a particle to settle to the bottom is equal to depth/velocity. Use the same range of particle sizes used in the preceding example. Note that you need to consider the units in the numerator and denominator. If the units of velocity are in cm/s what should the units of depth be?

**For r = 0, your calculation will be in error since a division by zero will occur.** Infinity is an upper limit, but not a practical or necessary one – especially for plotting! To explore the relationship between velocity and time variations as a function of particle radius just delete the value of time for r=0 and v=0. You can see what happens when you delete the first few values. The drop in velocity in time is quite rapid as the radius increases, so you may also want to see what happens if you plot the time axis on logarithmic scale. Why does this help?

**Also, remember to limit the maximum radius to 0.1 cm (i.e. 1 mm) in your plot.**
Checklist for the Quadratics and Settling Velocity problems
Chapter 3 Geomath

Assignment Checklist

Due : book problems 3.10 and 3.11

In 3.10 – you showed your derivation and included all steps. 5 points

In 3.11 you presented a brief derivation of the relationship \( \frac{v_1}{v_2} = \left( \frac{r_1}{r_2} \right)^2 \) (see page 54 text). 5 points

You also used the above relationship to calculate the settling time for a particle with radius 1mm, given that the settling time for a particle with radius 0.1mm is 10 days. You presented your calculations in organized form. 5 points

TOTAL 15 Points

Computer Problem for Today’s Efforts

To do list

1. plot of settling velocity versus particle radius. (3 points)
2. On a separate sheet of paper or on the plot page itself
   a. Explain how you will calculate lake depth (3 points)
3. Show your computations of the lake depth. (3 points)

Prepare a

4. plot of settling time versus particle radius for a 100 meter deep lake. (3 points)
5. Comment on how the plot of settling time compares to the plot of settling velocity. Think about this in the context of comments in class about the relationship of Stokes’ equation for velocity compared to the expression modified to show how time varies with particle radius. (3 points)

TOTAL 15 Points

Due date___________________________
Problem 3.11 Stokes’ law states that the viscosity at which a spherical particle suspended in a fluid settles is given by

\[ v = \frac{2(\rho_p - \rho_f)gr^2}{9\eta} \]

where \( v \) is the velocity of descent, \( \rho_p \) and \( \rho_f \) are the densities of particle and fluid respectively, \( g \) is the acceleration due to gravity, \( r \) is the particle radius and \( \eta \) is a property of the fluid known as \textit{viscosity}. Assuming that grains of different sizes have identical densities, show that the ratio of the settling velocities for two different grain sizes is

\[ \frac{v_1}{v_2} = \left(\frac{r_1}{r_2}\right)^2 \]

where \( v_1 \) and \( v_2 \) are the velocities for grains of radius \( r_1 \) and \( r_2 \) respectively.

- Now with that result in mind consider the following problem. If a grain of radius 0.1 mm, suspended in a lake takes 10 days to settle to the lake bottom, how long would it take a grain of radius 1mm?
Throughout the semester you’ve encountered a variety of mathematical relationships between various geologic variables such as age vs. depth, porosity vs. depth, earthquake magnitude and fault plane area, for example. Very often our data are derived from observations and the parameters defining these various relationships are useful to us as predictors in other situations. Geologists often deal with empirical data and must form quantitative models of geologic behavior based on these observations. These models can be used to make predictions and compare behavior observed in different locations, for different minerals, stratigraphic intervals, etc. The objective of the current lab is to provide you with some experience in the use of spreadsheet functions to derive model coefficients used to define linear models, exponential, power law and polynomial models of the sort we have been studying. The data is on the class common drive (H:\) in the FittingLabData folder. In side you will find the DepthAge data used previously along with other data sets that will be referenced in this lab guide. If you haven’t already, copy the FittingLabData folder over to your personal network (N:\) drive.

Example 1:
GET INTO EXCEL

Let's start with a simple linear relationship such as the one suggested between age and sediment thickness. Recall the data we worked with in an earlier lab that was taken from problem 2.11 in Waltham's text. In this problem, data shown in the table below were given for the Troll 3.1 well in the Norwegian North Sea. This is data you used before and should have on your G:\Drive.

<table>
<thead>
<tr>
<th>Depth</th>
<th>Age</th>
</tr>
</thead>
<tbody>
<tr>
<td>19.75</td>
<td>1490</td>
</tr>
<tr>
<td>407</td>
<td>10510</td>
</tr>
<tr>
<td>545</td>
<td>11160</td>
</tr>
<tr>
<td>825</td>
<td>11730</td>
</tr>
<tr>
<td>1158</td>
<td>12410</td>
</tr>
<tr>
<td>1454</td>
<td>12585</td>
</tr>
<tr>
<td>2060</td>
<td>13445</td>
</tr>
<tr>
<td>2263</td>
<td>14685</td>
</tr>
</tbody>
</table>


You can place column titles Depth and Age in columns A and B, cells A1 and B1.

Make a quick plot of the DepthAge data. We’ll use it to illustrate some curve fitting functionality.
Your plot should look like that shown at right.

Now this plot clearly demonstrates that the relationship between depth and age is non-linear. Earlier we recognized that two straight lines are needed to explain these data: one curve for depths less than 500 cm and another curve for depths greater than 500 cm. The graph shows us that sedimentation rate decreased abruptly about 10500 years ago.

In our previous analysis of this data we estimated the rate of change of age with change of depth from two points in each area.

\[
\frac{\Delta \text{Age}}{\Delta \text{Depth}} \quad \text{was computed from data points with depths extending from 19.75cm to 407cm and ages from 1490years to 10510 years. This yielded }
\]

\[
\frac{\Delta \text{Age}}{\Delta \text{Depth}} = \frac{9020}{387.25} = 23.29 \frac{\text{years}}{\text{cm}}.
\]

We did a similar analysis of the deeper data and obtained a more gradual age/depth relationship of \underline{__________________} (Remember how? Refer to your earlier work).

What we want to do now is use Excel to determine the slope of these data for us using all the data points over a specified range. We’ll limit our analysis to the deeper data since the shallow data consisting of only two points is a trivial case – (there’s only one possible answer!).

Excel will compute a trendline to the data. The computation minimizes the deviations of Age observations from the computed line. The line that Excel computes is the "best fit" line – trendline. But when Excel computes a trendline, it considers all the data points, whereas in our earlier lab we considered only the two end points and thus assumed that the other values fell along the general trend defined by those two points. Of course, in general, this may not be the case.
So begin by deleting the first row of data from your spreadsheet. We won’t need that data pair for this exercise. Delete the first set of x and y coordinates (19.75, 1490). When you delete that row, your spreadsheet and plot will look like that shown below. Note that the graph will be automatically updated to exclude the first data pair.

Change the age range to extend over the more limited 10,000 to 15,000 year time range (see below).
Several of the following examples (1 through 4) are presented solely for practice.

The specific homework assignment is discussed on pages 60-61. You have the option to pick one of two problems.

Fitting a curve to your data.

We assume, in this case that the relationship between Age and Depth follows a straight line over the 400cm to 2500cm depth range. The assumed relationship is linear in the form:

\[ \text{Age} = \text{Slope} \times \text{Depth} + \text{Intercept}. \]

To determine the coefficients of the line (i.e., the slope and intercept) - do the following:
Left click on the line to activate it; then right click the highlighted line and select the Trendline option in the drop down list that appears. This will bring up the following dialog window:
We will be coming back to this dialog window repeatedly in the following exercises. You can leave it open and go back and forth between different graphical elements. As you can see from the display there are a variety of trendline options. The one we want is the linear, so turn on the linear radio button. We also want to display the equation on the chart and to see the value of R-squared. R-squared basically gives you a measure of how closely the line fits the data or how good a job a line does in minimizing the sum of the squares of the differences of data points from the line. $R^2$ is referred to as the coefficient of determination and is a measure of the “goodness-of-fit.” The trendline, equation and $R^2$ should appear without closing the Trendline window. Your plot should look similar to that below.

![Graph with the equation $y = 1.906x + 9988$ and $R^2 = 0.953$]

You can increase the font of the Trendline equation and other properties of the Trendline Label.

What are the slope and intercept of the trendline?

The Slope = ____________

The Intercept = ____________

What is the sedimentation rate? ____________

Just for future reference, note that this linear trendline is often referred to as a linear regression line or best-fit line.
From the trendline equation, we see that the slope is 1.906 years per cm. This suggests that 2632 cm of sediment will accumulate, approximately, in a 5000 year time period.

Note that the fitted line doesn't pass through any of the data points, but is oriented in such a way as to follow the general trend of the data points and to minimize differences between locations of individual observations or values and the trendline. Thus, it is a “best-fit” line.

If sedimentation had continued at this rate, what would be the age of the surface exposure?

**Example 2:**

**Exponential Curve Fitting:**

Recall from the discussions in the text and lecture that porosity (in some areas) may decreases exponentially with depth according to the relationship $\phi = \phi_0 e^{-z/\lambda}$. Let's see if this general relationship holds for porosity depth observations made in the Stratton-Nordvick #4 well shown in the table below.

As illustrated in the plot (above), the porosity values drop off rapidly with increased depth. Porosity values approach zero below depths of 5 kilometers or so. Determine the "best-fit" parameters $\lambda$ and $\phi_0$ in the assumed exponential relationship stated above.
We do this just as we did above for the age-depth data. Activate the line (left click it) and then add a trend line. In this case, we choose the Exponential option (see below).

We’ll also display the best-fit equation and $R^2$ (coefficient of determination) on the chart. Close and then format the Trendline Label. Your plot should look similar to that below.

$$y = 0.503 e^{-0.45x}$$

$R^2 = 0.711$

$\phi_0 = 0.503$

$\lambda = 1/0.45$
The exponential decay relationship $\phi = ae^{bz}$ defines the general trend of the porosity depth variations quite well.

You get the values of the coefficients $\phi_0$ and $\lambda$ from the trendline equation! In this example, $a = \phi_0 = 0.5$, and $b = 1/\lambda$, so that $\lambda = 1/b = 1/0.45 \text{ or } 2.19 \text{km}$.

What do the constants in the equation defined as Y represent? The independent and dependant variables in the excel equations are always identified as $x$ and $y$.

The resultant model $\phi = 0.5e^{-z/2.19}$ could be used in nearby areas to predict the porosity at a given depth within a given reservoir interval. Note we have rounded off $\phi_0$ from 0.503 to 0.5 and that $\lambda = 2.19 \text{km}$.

There are two additional points to make: 1) the goodness of fit ($R^2$) is OK but not exceptional. 2) The intercept of the best fit line is 0.5 not 0.6. The goodness of fit is a little low because there is considerable scatter in the data. We could refit the trendline and require that the intercept equal 0.6.

To illustrate these two points a little differently, realize that the exponential form of this equation can be transformed into a straight line by taking the log of the porosity, where the natural log of $\phi = \phi_0 e^{-z/\lambda}$ yields $\ln(\phi) = \ln(\phi_0) - z/\lambda$. Let’s make this transformation graphically by changing the scaling on the $\phi$ axis to logarithmic rather than linear. To do that highlight the $\phi$ axis and RC >> Format Axis to bring up the following dialog. Check the logarithmic scale as shown below and you should see the plot auto scale to log $\phi$. 

![Format Axis dialog](image)
In this “Log-Linear” graphical display, the relationship is transformed into a straight line and, as expected, indicates that the natural log (or any log) of the porosity $\phi$ should vary linearly with depth $z$. Also note that the best-fit line is straight as it should be. The individual data points fall along this straight line but considerable scatter appears in the lower porosity values. In this format, it is easier to see where most of the error comes from. The data are “noisiest” at the greater depths. At greater depths the $\phi$ variations flatten out and approach zero. Also note that on the log scale these fluctuations are enhanced since on a logarithmic scale, variations from 0.001 to 0.01 will be as great as those from the much larger range 0.01 to 0.1.

Now, with your Format Axis window still open, uncheck the log scaling box. Click on the “best-fit” line and delete it. Now click on the data series (notice how the format box remains open and has shifted to Format Chart), RC & then Add Trendline. Check off the Exponential Radio button, set the value of the intercept to 0.6 and check off the Display Equation and $R^2$ boxes.

Note that although we haven’t reduced $R^2$ by very much, that we have much better fit at shallower depths where the most significant $\phi$ variations occur (0 to 4 km).
Example 3: Polynomial Fit

In Chapter 2, Waltham gives us the following Depth versus Temperature data for Earth. We discussed the relationship earlier in class when we introduced the general class of functions referred to as polynomials. You may want to refer back to those notes and to the text for additional discussion at this point.

<table>
<thead>
<tr>
<th>Depth (km)</th>
<th>Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>100</td>
<td>1150</td>
</tr>
<tr>
<td>400</td>
<td>1500</td>
</tr>
<tr>
<td>700</td>
<td>1900</td>
</tr>
<tr>
<td>2800</td>
<td>3700</td>
</tr>
<tr>
<td>5100</td>
<td>4300</td>
</tr>
<tr>
<td>6360</td>
<td>4300</td>
</tr>
</tbody>
</table>

Open the DepthT.xls file in the FittingLab Folder and generate a plot.

Waltham considers two possible functional relationships between $T$ and $z$:

1) \[ T = a_2 z^2 + a_1 z + a_0, \] and
2) \[ T = a_4 z^4 + a_3 z^3 + a_2 z^2 + a_1 z + a. \]

These are two polynomials. 1) is a second order polynomial (the quadratic) and 2) is a 4th order polynomial.

After you have your plot formatted and labeled, click on the data series and RC to get the Data Series Trendline dialog window. Note that in the Trend/Regression type list, there is a polynomial fitting option. When you select the Polynomial it will probably default to order 2 and your graphical display window will automatically be updated to include the best-fit quadratic. Select “Display Equation on Chart” and “Display R-squared Value on Chart.”

Relocate and format your equation and $R^2$ text box. When done, your display window should look something like that shown on the next page.

You may have to format trendline label (RC on label box edge) and select Format Trendline Label. Change type to number and put in 5 decimal places. Vary as needed. The number of decimal places depends on the data set.
Now **repeat** the above, but use a **4th order polynomial**. If you’ve left your format window open, you can get back to the Trendline dialog just by clicking on the trendline in your plot. Change order 2 to 4; delete your equation and $R^2$ text box; click on the trendline again; check off the show equation and $R^2$ check boxes and you should have a plot that looks like that below with updated equation and $R^2$.

Now **repeat** the above, but use a **4th order polynomial**. If you’ve left your format window open, you can get back to the Trendline dialog just by clicking on the trendline in your plot. Change order 2 to 4; delete your equation and $R^2$ text box; click on the trendline again; check off the show equation and $R^2$ check boxes and you should have a plot that looks like that below with updated equation and $R^2$.

Note that the value of $R^2$ has increased slightly since the higher order fit is a better one.
The parameters and general shape of our result is different from that of Waltham's. If you look at Waltham's data plot in Figure 2.8, it appears that he excluded the first data point from consideration in the fitting process. This is easily done. If you delete the Depth and Temperature values in the first line, the plot will automatically be updated. We get a result that agrees much better with Waltham's (see below).

![Temperature vs. Depth in the Earth's Interior](image)

**Example 4:**
**Power Laws**

Notice that the coefficients defining a power law relationship such as $a$ and $b$ in the equation $N = ar^b$ could be estimated using the **Math, Fitting, Power Law** option. Give it a try. Use the following table of data (file in the Fitting Lab folder).

<table>
<thead>
<tr>
<th>R</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10000</td>
</tr>
<tr>
<td>2</td>
<td>3789.29</td>
</tr>
<tr>
<td>4</td>
<td>1435.87</td>
</tr>
<tr>
<td>8</td>
<td>544.094</td>
</tr>
<tr>
<td>16</td>
<td>206.173</td>
</tr>
<tr>
<td>32</td>
<td>78.129</td>
</tr>
<tr>
<td>64</td>
<td>29.604</td>
</tr>
<tr>
<td>128</td>
<td>11.22</td>
</tr>
<tr>
<td>256</td>
<td>4.25</td>
</tr>
</tbody>
</table>

You will find that $a$ is 10000 and $b$ is -1.4.
**Homework assignment:**

Do one of the following two problems.

**Option I:** This problem examines another exponential process. \( t = t_0 e^{-x/X} \) (see discussion of in problem 4.5 in the text), which defines a relationship between thickness of the bottomset beds (t) at the foot of a delta (see figure below) and the distance from the onset of the bottomset bed (x); \( t_0 \) and \( X \) are constants.

![Diagram of internal bedding geometries in a simple delta.](image)

*Figure 2.9: Internal bedding geometries in a simple delta.*

Following the format of Example 2a, determine the constants \( t_0 \) and \( X \) given the following measurements of bottomset bet thickness (t) and distance (x).

Follow the procedures illustrated for example 2a and determine the constants \( t_0 \) and \( X \) for the bottomset bed thickness/distance measurements tabulated above. Estimate the thickness of a bottomset bed at a distance 4.5 km from the bed start.

The data listed in the table at right can be found on your common (H) drive in the file *BottomSetData.xls*

<table>
<thead>
<tr>
<th>( X ) (km)</th>
<th>( t ) (meters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>20.354</td>
</tr>
<tr>
<td>0.2</td>
<td>16.545</td>
</tr>
<tr>
<td>0.4</td>
<td>11.204</td>
</tr>
<tr>
<td>0.6</td>
<td>11.072</td>
</tr>
<tr>
<td>0.8</td>
<td>5.464</td>
</tr>
<tr>
<td>1</td>
<td>4.221</td>
</tr>
<tr>
<td>1.2</td>
<td>3.471</td>
</tr>
<tr>
<td>1.4</td>
<td>3.124</td>
</tr>
<tr>
<td>1.6</td>
<td>3.416</td>
</tr>
<tr>
<td>1.8</td>
<td>0.509</td>
</tr>
<tr>
<td>2</td>
<td>1.550</td>
</tr>
<tr>
<td>2.2</td>
<td>1.788</td>
</tr>
<tr>
<td>2.4</td>
<td>1.980</td>
</tr>
<tr>
<td>2.6</td>
<td>1.187</td>
</tr>
<tr>
<td>2.8</td>
<td>0.397</td>
</tr>
<tr>
<td>3</td>
<td>1.887</td>
</tr>
<tr>
<td>3.2</td>
<td>0.750</td>
</tr>
<tr>
<td>3.4</td>
<td>0.830</td>
</tr>
<tr>
<td>3.6</td>
<td>0.801</td>
</tr>
<tr>
<td>3.8</td>
<td>0.773</td>
</tr>
<tr>
<td>4</td>
<td>0.020</td>
</tr>
</tbody>
</table>

Option I - Presentation format:
1. State the problem (2 points)
2. Include your plot of \( t \) vs. \( x \) (8 points properly labeled with title and name)
3. Include a plot of the "best fit" line in the above plot along with its equation (4 points)
4. Explicitly state what the derived value of \( t_0 \) and \( X \) correspond to (3 points)
5. Show your calculations of \( t \) at \( x = 4.5 \) km (5 points)
6. Explicitly state your result (3 points)

Total of 25 possible points
Option II:
You are given the following earthquake frequency/magnitude observations for an area in central Japan.

<table>
<thead>
<tr>
<th>Magnitude (m)</th>
<th>N (M&gt;m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>606</td>
</tr>
<tr>
<td>2.75</td>
<td>274</td>
</tr>
<tr>
<td>3</td>
<td>175</td>
</tr>
<tr>
<td>3.25</td>
<td>96</td>
</tr>
<tr>
<td>3.5</td>
<td>60</td>
</tr>
<tr>
<td>3.75</td>
<td>33</td>
</tr>
<tr>
<td>4</td>
<td>26</td>
</tr>
<tr>
<td>4.25</td>
<td>15</td>
</tr>
<tr>
<td>4.5</td>
<td>10</td>
</tr>
<tr>
<td>4.75</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
</tr>
</tbody>
</table>

Estimate the parameters $b$ and $a$ in the Gutenberg-Richter relationship:

$$\log N = -bm + a$$

Given these estimates of $b$ and $a$, determine how frequently a magnitude 6 or greater earthquake can be expected in this area. The answer should be expressed in terms of the time period in which one magnitude 6 or greater earthquake can be expected to occur. In other words, your analysis will yield the time in years during which you would expect to see at least one magnitude 6 or greater earthquake. What is that time interval? You already solved this problem in an earlier assignment.

Hint - let $m = 6$ in the above equation.

Option II Presentation format:
1. State the problem (2 points)
2. Compute the log(N) and plot log(N) versus m (8 points properly labeled with title and name)
3. Include a plot of the "best fit" line with equation (4 points)
4. Explicitly state what $b$ and $a$ are in the formula $\log N = -bm + a$ (3 points)
5. Present your calculations of the recurrence time during which one magnitude 6 or greater earthquake is likely to occur (5 points)
6. Explicitly state your result (3 points)

Bring questions to class March 2nd (section 2), March 3rd (Section 1).

Due Date March 5th in my mailbox by noon. Note that these problems will provide review of several concepts relevant to the midterm exam.
The probability of making errors increases with the complexity of computations especially when numerous organizational and computational steps are required to arrive at the solution. The estimation of maximum sustained yield from pump test data is an excellent example of numerical analysis that involves several transforms of the raw field data, combined with logarithmic and regression line computations, and a mixture of different units in which the input and output data can be reported. An error at any stage in this complicated process will yield an incorrect answer. In this lab computations are structured to help you minimize the chances of making simple errors in a long stream of computational steps.

While the initial key to success is to thoroughly understand the background for the mathematical process that is being performed, Dr. Rauch’s hydrogeology course will introduce you to the underlying theory associated with this example. This exercise is taken from one of Dr. Rauch’s class handouts. The brief discussions presented in our class provide a basic summary of the main ideas involved in this procedure and of the pertinent measured and calculated quantities. The exercise will hopefully, better prepare you to handle problems encountered in his or other hydrogeology classes should you take them in the future.

Of equal importance, is the need to summarize the steps taken in the analysis, the definition of terms, the list of the equations you will be working with, and the set up and sequence of calculations to be performed.

**Definition of Terms**

**Q** - In a pump test, water is removed from a well at a constant rate \( Q \), the pumping rate. This rate is usually reported in gallons per minute (gpm). \( Q \) is also used later to represent the maximum sustained yield, which will be one of your last computations. You will often encounter situations where the same symbol is used to represent different variables, so you just have to keep the application clearly in mind to avoid confusion.

\( t \) – The time since well pumping was initiated and \( t' \) - the time since pumping ceased.

**\( s_u \)** - The basic data collected in a pump test are the depth to the water surface (\( d \)) in the well and the duration time (\( t \)) of pumping. The value \( d \), however, needs to be transformed into a number related to the depth below the undisturbed depth of the water table in the well prior to pumping. This depth is referred to as the static water depth (\( d_s \)) and \( d_s \) is subtracted from \( d \) to obtain the drawdown, \( s_u \). \((s_u = d - d_s)\)

**\( m \)** – the initial saturated thickness of the aquifer.

**\( s_a \)** – The drawdown is further transformed into a quantity referred to as the equivalent (or corrected) drawdown \( (s_a) \). \( s_a = s_u - s_u^2/2m \).
During the pump test, well pumping is maintained for a certain period of time and the characteristics of the water table “drawdown” are observed. The variation of drawdown with time provides information about the transmissivity and hydraulic conductivity of the water reservoir. However, transmissivity and hydraulic conductivity can also be estimated from the recovery of the water table following the cessation of pumping. Hence, the independent variable, time, is a variable that is relative to the process being studied – drawdown or recovery. The characteristics of recovery are followed in time relative to the time when pumping stopped. This time is referred to as \( t' \). In addition, water table recovery is not simply a function of time since pumping stopped, but is related in a complex way to the duration of pumping. Recovery after a long period of pumping will have different characteristics than the recovery after a shorter period of pumping. The ratio of the times \( t/t' \) provides a measure of this relative time – the time of recovery relative to the time since pumping began.

\[ \log_{10}(t/t') \] – The relation between \( s_a \) and \( t' \) during the recovery phase of the pump experiment is logarithmic. \( s_a \propto \log_{10}(t/t') \). Hence the base 10 logarithm of the ratio \( t/t' \) must be computed.

As evident from the above discussion, estimates of transmissivity (\( T \)) and hydraulic conductivity (\( K \)) can be made from the pumping and recovery observations of drawdown.

**Key Mathematical Relationships**

The mathematical relationships of \( T \) to \( s \) and \( t \) are shown below.

**Equations for \( T \) and \( K \) derived from pumping stage observations**

\[
T = 2.303Q\left(\log_{10} t_2 - \log_{10} t_1\right)/4\pi(s_2 - s_1)[(\text{gal/min/ft})] \tag{1}
\]

since the slope = \( \frac{(s_2 - s_1)}{(\log_{10} t_2 - \log_{10} t_1)} \) the above equation becomes

\[
T = 2.303Q/4\pi(slope)[(\text{gal/min/ft})] \tag{2}
\]

In these equations \( s_1 \) and \( s_2 \) are corrected drawdowns (\( s_a \)).

The units assumed in formula (1) and (2) are \( \text{gal/min/ft} \). If you wish to report your results in \( \text{ft}^3/\text{d/ft} \) then multiply \( Q \) by 1440 \( \text{min/day} \) to get \( Q \) in gallons/day and divide \( Q \) (in gallons/day) by 7.48 \( \text{gal/ft}^3 \) to get \( Q \) in units of \( \text{ft}^3/\text{day} \). If you measured drawdown in meters, then you would have to convert feet to meters using the conversion factor 3.281 feet/meter.

Equation 1) or 2) above (in units of (\text{gal/min/ft})) can be computed directly in units of \( \text{gal/d/ft} \) using the following equation

\[
T = 264Q/(slope) \tag{3} \quad \text{[(gal/d)/ft]}
\]

where \( 264 = 2.303 \times 1440\text{min/day}/4\pi \).

We could make the further conversion of gallons to cubic feet and write \( T \) as
\[ T = 35.3Q/\text{(slope)} \] [(ft\(^3\)/d)/ft] (where Q is in gallons/minute)  

Perhaps the greatest potential for error in these calculations is in the units conversion process. In the following example, the units of Q (the pumping rate) are gallons/minute; the units of \( s \) are feet; and the units of time are seconds. The result for K will be requested in (gpd)/ft\(^2\).

The hydraulic conductivity K is

\[ K = \frac{T}{m} \]

and is usually expressed in units of feet or meters per day.

**Equations for T and K derived from recovery stage observations**

The recovery stage estimates of T are almost identical. However, the time variable is \( t/t' \), and the plotted data form which the slope is derived are \( s \) vs. \( \log(t/t') \). Hence we rewrite Equation 1) above as

\[ T = 2.303Q(\log_{10} t_2 / t_1' - \log_{10} t_1 / t_1') / 4\pi(s_2 - s_1) \]

**Maximum long-term sustained yield**

The long-term sustained yield (Q) is written as

\[ Q = K(H^2 - h^2)/[1055\log_{10}(R/r)] \]

The expression contains some undefined variables. H is the static saturated aquifer thickness above the well bottom. \( H = m \) so nothing new here. h is the height of the water column above well bottom during maximum sustained pumping. R is the radius of the steady state pumping cone of depression in the water table, and r is the well radius below the casing (outside radius). K is still the hydraulic conductivity.

In practice two estimates of Q are made using K's derived from the pumping and recovery phases of the pump test.

**Background on the PumpTest Data for this Lab Exercise**

The following data are taken from Dr. Rauch's handout *Aquifer Pumping Tests and Well Hydraulics*.

**HYDROGEOLOGY WELL PUMP TEST RESULTS**

On Sept., 27, 1995 a pump test was conducted on the new well drilled earlier in September. Henry Rauch of W.V.U. conducted this test, which was witnessed by Richard Cato and two of his assistants. As a result of this test, both Rauch and Cato concluded that this well is very adequate for the W.V.U. Forestry Division's needs as a water supply.

The well static water depth was first measured and was found to be 74.11 feet below ground surface; then the well was pumped at 13.8 gallons per minute for one hour,
resulting in a drawdown of the water level by 54.21 feet, to a new depth of 130.42 feet. Many measurements of pumping rate and water level depth were made during this pumping test phase. The pump was then shut off and the water level depth was again measured many times for the next hour during which the water level recovered to a depth of 96.03 feet, representing a 63% recovery in water level compared to pre-pumping level. Based on the measured data, the estimated maximum sustained yield of the well was calculated, using aquifer transmissivity (T) and hydraulic conductivity (K) values obtained from both the Jacob straight-line time-drawdown method for the well pumping test phase (Cooper and Jacob, 1946; Jacob, 1950), and the Jacob single well recovery test method (Jacob, 1963). The T and K obtained were then used to calculate maximum sustained well yield (Q) using the equilibrium well formula given by Driscoll (1986). It was assumed that the tested aquifer is unconfined, so that appropriate formulas and data corrections were applied for unconfined aquifers.

**Pump Test Data**

In the table of pump test data below, clock-time has not been used. The reported times are in minutes since pumping began. The following data are provided on shared or common drives.

<table>
<thead>
<tr>
<th>Observation number</th>
<th>Time (minutes)</th>
<th>Water depth (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>76.21</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>89.14</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>93.92</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>96.78</td>
</tr>
<tr>
<td>5</td>
<td>12</td>
<td>99.42</td>
</tr>
<tr>
<td>6</td>
<td>15</td>
<td>102.63</td>
</tr>
<tr>
<td>7</td>
<td>18</td>
<td>105.63</td>
</tr>
<tr>
<td>8</td>
<td>21</td>
<td>108.63</td>
</tr>
<tr>
<td>9</td>
<td>25</td>
<td>111.66</td>
</tr>
<tr>
<td>10</td>
<td>29</td>
<td>115.19</td>
</tr>
<tr>
<td>11</td>
<td>33</td>
<td>117.87</td>
</tr>
<tr>
<td>12</td>
<td>36</td>
<td>119.69</td>
</tr>
<tr>
<td>13</td>
<td>41</td>
<td>122.41</td>
</tr>
<tr>
<td>14</td>
<td>43</td>
<td>123.47</td>
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<tr>
<td>15</td>
<td>47</td>
<td>125.42</td>
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<td>16</td>
<td>50</td>
<td>126.79</td>
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<td>17</td>
<td>55</td>
<td>128.5</td>
</tr>
<tr>
<td>18</td>
<td>58</td>
<td>129.83</td>
</tr>
<tr>
<td>19</td>
<td>60</td>
<td>130.42</td>
</tr>
<tr>
<td>20</td>
<td>63</td>
<td>124.89</td>
</tr>
<tr>
<td>21</td>
<td>69</td>
<td>120.12</td>
</tr>
<tr>
<td>22</td>
<td>73</td>
<td>116.92</td>
</tr>
<tr>
<td>23</td>
<td>77</td>
<td>114.41</td>
</tr>
<tr>
<td>24</td>
<td>81</td>
<td>112.03</td>
</tr>
<tr>
<td>25</td>
<td>91</td>
<td>106.24</td>
</tr>
<tr>
<td>26</td>
<td>101</td>
<td>102.3</td>
</tr>
<tr>
<td>27</td>
<td>109</td>
<td>99.31</td>
</tr>
<tr>
<td>28</td>
<td>116</td>
<td>97.2</td>
</tr>
<tr>
<td>29</td>
<td>120</td>
<td>96.03</td>
</tr>
</tbody>
</table>
Get into Excel
Excel functionality allows us layout our computations in an organized and easy to follow manner. The Excel computations can also be set up to allow interactive updating of the calculations with changes of input parameters.

To begin with, enter the times (in minutes) and depths (in feet) into columns C and D. Keep two or three rows open at the top of these columns to add column labels as shown in the Figure below. We are leaving columns A and B open to list variables and their values. In column E make the conversion from minutes to days. Starting in row 4 cell E4 type in the formula: =c4/1440.

Before we go further add a problem title and the input variables in column A (see entries below). Enter corresponding values in column B.
Now that you have the various input parameters available for reference, you can compute the corrected depth. Make computations of the corrected depth in Column F. In column F, Row 4 (as set up below), enter the equation =D4-BSB4; this yields values of water depth relative to the static depth of the water table. Remember the $ signs prevent the reference to column (B) and row (4) (the cell containing the value of the static water depth) from changing when the formula is copied. Copy this cell into all rows down to row 32. Click on an arbitrary cell in column F and confirm that the D4 reference changes with the row and that the references to cell B4 do not change, since the B4 reference has the $ signs before the B and the 4.

Next, in column G, calculate the logarithm of time (in days). In row 5 enter =LOG(E5); then copy into remaining rows. In column F, enter the formula for the corrected drawdown: =F4-F4^2/(2*$B$3). $B$3 designates a static reference to the saturated thickness of 229 feet. Copy this formula into the remaining rows – down to row 32. Your spreadsheet should look like that below.

Generate a plot of the LogTime (column G) versus Corrected Drawdown (column H) data. Label the Pumping and Recovery Stages as shown below. Also note the region of linear drawdown near the end of the pumping and recovery stages.
Do you remember why the **Log Time** and **Corrected Drawdown** columns are highlighted from 41 to 60 minutes on page 64? The drawdown from 41 to 60 minutes corresponds to the drawdown behavior at the end of the drawdown phase of the pump test. The highlighted points correspond to the 7 data points along the line at the end of the drawdown phase shown in the figure above. After 60 minutes of pumping the pump was turned off and the water table began to recover.

The recovery times extend from 60 to 120 minutes and the recovery **duration** extends from 0 to 60 minutes. Include these two columns of data in Columns I and J and take the log of their ratio (log of the recovery time over the recovery duration), i.e. log(t/t’) in column K. See Excel screen capture on the next page for reference to your work.
Your Excel spreadsheet should look similar to that shown below. Note that the drawdown and recovery phase data have been highlighted (yellow and blue, respectively), and the corrected drawdowns during the recovery phase have been copied into column L.

The transmissivity and conductivity revealed by the drawdown phase results have been calculated for you (next page). I’ve listed the results in various units in the Excel display (below). The Excel formulas are shown on page 70. Repeat these calculations and cross check with the results shown below.
Computing the slope
Excel will calculate the slope of the best fit line to a specified range of data. This computation is more direct and does not require adding a trendline to your graph. Excel has numerous built in functions commonly used in a variety of applications. The function list is something worth spending a little time exploring. You may find some useful shortcut approaches to computation along with additional things to do with your data that are already automated for you. To access the Excel function list click on the formulas tab in the ribbon. At far left you will see an $f_x$ insert function button.

Click on this to get the following drop down window.

![Insert Function Window](image)

In the categories drop down you’ll find statistical, math and trig functions listed along with a variety of different categories of functions. If you are looking for something you can’t find, you could use the general search window. For example, you could just type slope into the “search for a function” window, which would turn up the function list shown below.
For this exercise, we will want to use the SLOPE function at the top of the list. The format of the Slope function is shown just below the “select a function” window – SLOPE(known_y’s,known_x’s).

In the highlighted Drawdown Slope calculation box (below) you’ll see the slope function used to compute the slope from the appropriate drawdown Log Time and Corrected Drawdown values highlighted in columns G and H.

Slope(H16:H22,G16:G22) identifies the specific range of values for which to compute the slope. This is very convenient since we don’t have to separate out the Log Times and Corrected Drawdowns of interest into separate columns. If correctly entered, the value of the slope should appear (see cell value B18 above).
**In summary**: The slope is computed using the slope function. In the figure above, the drawdown slope computation is highlighted:

\[
\text{Drawdown Slope} = \text{slope}(H16:H22,G16:G22)
\]

In this expression, \(H16:H22\) defines the Corrected Drawdown range used for this analysis in column H and \(G16:G22\) defines the range of corresponding Log Times. Note that the inputs in the slope function are \text{slope}(yrange,xrange). During the drawdown phase you should obtain a slope of 37.69 feet.

**The transmissivity \(T\)** is listed in three sets of units computed using equation

2) \(T = 2.303Q / 4\pi(\text{slope})\) [\((\text{gal/min)/ft}\)]

3) \(T = 264Q / (\text{slope})\) which has units of gal/d/ft or from equation

4) \(T = 35.3Q / (\text{slope})\), which has units of ft\(^3\)/d/ft.

We were given \(Q\) above as 13.8 gallons/minute. Remember that equations 3) and 4) have been set up so that we can use \(Q\) in gallons/minute and slope in feet, so no additional units conversions are necessary. Plugging values for \(Q\) and the \text{slope} (37.69 feet) into equation 3), we get \(T = 96.66(\text{gal/d)/ft})). Plugging into equation 4), we get \(T=12.92\) (ft\(^3\)/d)/ft.

5) The hydraulic conductivity \(K\) (=\(T/m\)) (Equation 5) is just \(12.92/229\) or 0.057 ft/d or in units of gpd/ft\(^2\) \(K=96.66/229\) or just 0.42 gpd/ft\(^2\).

7) The maximum sustained yield is calculated using Equation 7) -

\[
Q = K(H^2 - h^2)/[1055\log_{10}(R/r)]
\]

Equation 7) provides maximum sustained yield in units of gpm, and assumes that \(K\) is reported in units of gpd/ft\(^2\), hence we use \(K =0.42\text{gpd/ft}^2\) as derived above. Recall that \(H = m = 229\) feet. \(h\) is the assumed height of the water column above the base of the well during maximum sustained pumping, which was assumed by Dr. Rauch to be 20 ft; \(R\) is the radius of the steady state pumping cone of depression in the water table. Values of \(R\) are actually unknown, but based on experience are believed to fall between 100 and 1000 feet. Dr. Rauch calculate \(Q\) using three values for \(R\); \(R=100, R=300\) and \(R=1000\) feet. \(r\) is the well radius below the casing, which at this site was 3.5 inches or 0.292 feet.

\[
Q = 0.422(229^2 - 20^2)/[1055\log_{10}(300/0.292)] = 6.91\text{gpm} \text{ using an R of 300 ft.}
\]

For illustration, we will restrict ourselves to one calculation; however, separate calculations for \(R\) of 100 feet and 1000 feet provide perspective on the possible range of sustained flow that might result. Dr. Rauch reports sustained flows that range between 5.9 and 8.2 gpm for \(K\) estimated from the pumping phase drawdown response.

The excel formulas used to compute the drawdown phase values are shown in the following figure.
The estimates of T & K from the recovery stage data are left for you to complete.

Estimates made from the pumping and recovery phases of drawdown observations provide slightly different values of T and K. The true transmissivity and hydraulic conductivity probably lies somewhere between these two estimates. Computations of T and K are usually made from both the drawdown and recovery phases to provide perspective on the range of possible sustained yields that may occur over the actual lifetime of a particular well.

In the remainder of this lab, we estimate T for the recovery phase using Equation 6) where \( T = 2.303Q(\log_{10} t_2 / t_1 \, - \, \log_{10} t_1 / t_1') / 4\pi(s_2 - s_1) \). Note that in this equation the slope is derived from s versus log\(_{10}(t/t')\) data. Hence your first task will be to construct a plot of s versus log\(_{10}(t/t')\) obtained from water depths recorded following the cessation of pumping. The log \( t/t' \) and corrected drawdown data from the recovery stage were placed in columns K and L in the illustration above (see page \( \) ).
Recovery Phase Analysis

The remainder of the exercise requires that you make independent computation of the Transmissivity, Conductivity and Maximum Sustained Yield from the recovery phase behavior. You could structure your spreadsheet layout as shown below.
Although the computed results are nicely tabulated in the Excel spreadsheet, don’t forget that you are still expected to show individual calculations of T, K, and Q.

Assignment Check List

1. Hand in a plot of the Pump Test data (see page 64). Label the pumping and recovery phase portions of the data. Place a caption on the figure to indicate what the figure represents. Place your name in the plot title.
2. Summarize results (T, K, and Q) obtained from the pumping stage.
3. Prepare and hand in a plot of the recovery phase data (see page 72). Note the slope you obtained. Add appropriate labels and caption to the figure for clarification. Place your name in the plot title.
4. Determine T in units of [(gal/d)/ft] (use equation 3 above). Show your calculations and state what it is you are presenting.
5. Compute Hydraulic conductivity K in units of gpd/ft². Show your calculations and state that this is what you are doing.
6. Solve for Q (gpm) using equation 7 for R= 300 feet. Show calculations and state results.
7. Clearly show your calculations for transmissivity, conductivity and sustained yield.
8. In summary, list your results and clearly indicate the values obtained for transmissivity, conductivity, and sustained yield. Compare results obtained from the pumping and recovery stages of the test.
9. Always present your results in an organized fashion; clearly label and reference figures in your work statements.
1. Given a triangle with hypotenuse of length 1, and sides of length x and y as shown below, define the various trigonometric functions;

\[
\begin{align*}
\sin(\delta) &= \frac{y}{1} \\
\cos(\delta) &= \frac{x}{1} \\
\tan(\delta) &= \frac{y}{x} \\
\cot(\delta) &= \frac{x}{y} \\
\sec(\delta) &= \frac{1}{\cos(\delta)} \\
\csc(\delta) &= \frac{1}{\sin(\delta)}
\end{align*}
\]

2. Convert the following angles from degrees to radians:
   a) 30 degrees
   b) 45 degrees

3. Convert the following angles from radians to degrees:
   a) 0.5
   b) 1.2

4. At the base of a cliff you are standing on top of geological Unit A. The cliff face is formed along a normal fault (nearly vertical). The top of Unit A is also exposed at the top of the cliff face. You walk a distance \( x = 200 \) feet away from the fault scarp. Looking back toward the cliff, you use your Brunton and measure and note that the top of the cliff is \( 23^\circ \) above the horizon. What is the offset along this fault?
5. In the example illustrated below, a stream erodes less resistant fault gauge leaving an exposed fault scarp on the distant bank. You are unable to traverse the stream or make your way to the top of the exposure. Using your Brunton compass, you stand on the left edge of the stream and measure the angle (a) formed by the top of the cliff and the horizontal. You walk to the left 175 feet and measure angle (b). Angle a measure 31° and angle b, 19°. How can you determine the cliff height? What is the width of the stream?

6. Using the diagram and results from problem 2 assume the cliff face is capped by a resistant sandstone layer. The sandstone is underlain by less resistant shale which has been eroded by the stream. 38.4 feet of sandstone are exposed on the upper cliff face. The shale is exposed on the left bank of the stream and has a dip of 30°. What is the thickness of the shale bed?
7. The three point problem uses elevations measured at three points on a stratigraphic surface to determine the strike and dip of that surface. The elevations and locations of these points can be measured at the surface or, more likely, in the borehole. In the following problem, you have data from three boreholes (located in the map below) indicating subsea depths to the top of the Oriskany Sandstone as shown.

Determine the strike and dip of the Oriskany surface.

Take Home Problem

8. Solve Question 5.9 in Waltham given as: Two exposures, 500 m apart, are 400 m and 200 m, respectively, from a church. Calculate using equations 5.17, 5.19, and 5.20, the angles contained by the triangle defined by the two exposures and the church. Use Waltham’s Excel file Trig.xls (see http://www.gl.rhbnc.ac.uk/~dave/sheets/Trig.xls or pick up the files from the H:\Drive) to confirm your result.
In-Class Exercise on Statistics
Generating histograms and Computing Probabilities with PSIPlot

The following computer activities are designed to accompany today’s lecture. The exercise uses data presented by Waltham in Chapter 7, Table 7.1. The data consist of the masses of pebbles collected at a beach. This dataset - `pebmass.pdw` – will be provided for you in my shared directory or on the H:\drive. Copy that file to your G:/Drive.

**I. Descriptive Statistics:**

In our discussion we describe various statistical properties of the sample, such as its mean, variance and standard deviation. PSIPlot can be easily used to generate the descriptive statistics of your data. To do that, first -

**OPEN PSIPlot and then from FILE OPEN select the file pebmass.pdw.**

Then click on **Data, Descriptive Statistics** (see below).

The following window should appear

You will have only one variable in your column list so just **click OK.**
The following window will appear:

Note that the mean, standard deviation, variance and other statistical properties are summarized in this table for the list of pebble masses.

<table>
<thead>
<tr>
<th>COLUMN NAME:</th>
<th>MASS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of rows:</td>
<td>100</td>
</tr>
<tr>
<td>Number of valid points:</td>
<td>100</td>
</tr>
<tr>
<td>Number of missing points:</td>
<td>0</td>
</tr>
<tr>
<td>Number of negative values:</td>
<td>0</td>
</tr>
<tr>
<td>Number of positive values:</td>
<td>100</td>
</tr>
<tr>
<td>Number of zero values:</td>
<td>0</td>
</tr>
<tr>
<td>Minimum value:</td>
<td>224.00000000</td>
</tr>
<tr>
<td>Maximum value:</td>
<td>454.00000000</td>
</tr>
<tr>
<td>Inter range value:</td>
<td>230.00000000</td>
</tr>
<tr>
<td>Median:</td>
<td>352.50000000</td>
</tr>
<tr>
<td>Sum of row value:</td>
<td>35018.00000000</td>
</tr>
<tr>
<td>Sum of absolute value:</td>
<td>35018.00000000</td>
</tr>
<tr>
<td>Arithmetic mean:</td>
<td>350.15000000</td>
</tr>
<tr>
<td>Geometric mean:</td>
<td>346.69461326</td>
</tr>
<tr>
<td>Quadratic mean:</td>
<td>353.37806949</td>
</tr>
<tr>
<td>Harmonic mean:</td>
<td>343.33964895</td>
</tr>
<tr>
<td>Absolute mean:</td>
<td>350.18000000</td>
</tr>
<tr>
<td>Sum of squares:</td>
<td>12487605.0000</td>
</tr>
<tr>
<td>Variance:</td>
<td>272.75515152</td>
</tr>
<tr>
<td>Standard deviation:</td>
<td>47.67342186</td>
</tr>
<tr>
<td>Absolute deviation:</td>
<td>38.37640000</td>
</tr>
<tr>
<td>Standard error:</td>
<td>4.75734219</td>
</tr>
</tbody>
</table>

II. Examining the Distribution of Pebble Mass using the Histogram:

The instructions below will take you through the generation and plotting of a histogram using PSIPlot. The histogram provides a graphical display of the distribution of your data.

In order to generate a histogram you have to subdivide your data. These subdivisions or bins usually correspond to intervals of data having the same size. First, let’s sort the data into ascending order. Click on the Column button (see below) on the top menu bar, click on sort, and then select ascending in the sort window. Click Add to transfer the column over into to the sorting order window. Then, click OK. Mass will now be sorted in increasing order, from the smallest to largest mass. What are the limits (maximum and minimum values) of the pebble mass data?

Minimum Mass = ______________; the Maximum Mass = ______________
Based on the range, we will adopt the 50 gram subdivisions or “bins.” That Waltham uses to subdivide the pebble mass data. In column 3 type in 200 (in cell 1), 250, 300, … etc to 500. Label that column BIN (or use the fill selection option – see right).

The histogram is just a bar-plot of the number of data points that fall in each subdivision or bin. In the “old-days,” one just sat down and counted the number of data points falling into each subdivision. PSIPlot will do this for you. Create the histogram data by going to Plot - 2D Special

Click on Plot - 2D Special - Histogram
The following window opens. **Highlight** the Mass column, Click **Bin Column as Interval** checkbox with 10 intervals as the default.

Note that instead of using the Bin column, we could have specified the lower and upper limits and values and number of intervals.

Click OK - You should get the following plot called a histogram.

We did not have to sort the values first, but the relationship of the sorted data to the histogram is a more direct one. As you scroll down through the column of sorted masses you see that there are a much greater number of values with masses in the 340, 350, 360 gm range. The frequency of occurrence is greater over this range than over others.
Frequency Count
PsiPlot does not automatically return the frequency count (number of samples falling in the interval). But there is a relatively easy way to obtain this information. Start by clicking on Column and then Frequency Count. Note that the following window will appear. You can type in the individual ranges used to construct the histogram and quickly obtain the number of sample values falling in that interval (see below for interval extending from masses of 200 grams to 250 grams).

<table>
<thead>
<tr>
<th>Range (g)</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>201-250</td>
<td>2</td>
</tr>
<tr>
<td>251-300</td>
<td>12</td>
</tr>
<tr>
<td>301-350</td>
<td>35</td>
</tr>
<tr>
<td>351-400</td>
<td>36</td>
</tr>
<tr>
<td>401-450</td>
<td>14</td>
</tr>
<tr>
<td>451-500</td>
<td>1</td>
</tr>
</tbody>
</table>

See Table 7.3 of Waltham.

The number of occurrences observed in each of these intervals, divided by the total number of observations in the sample represents the probability of finding a pebble with a mass in that 50 gram interval in that area of the beach. We are assuming that the sample is representative of the distribution of the pebbles in that area.
Construct two additional columns, BinC (bin centers) and PROB (see below), in your worksheet.

<table>
<thead>
<tr>
<th>mass</th>
<th>BinC</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>374.000</td>
<td>225.000</td>
</tr>
<tr>
<td>2</td>
<td>389.000</td>
<td>275.000</td>
</tr>
<tr>
<td>3</td>
<td>358.000</td>
<td>325.000</td>
</tr>
<tr>
<td>4</td>
<td>395.000</td>
<td>375.000</td>
</tr>
<tr>
<td>5</td>
<td>371.000</td>
<td>425.000</td>
</tr>
<tr>
<td>6</td>
<td>334.000</td>
<td>475.000</td>
</tr>
<tr>
<td>7</td>
<td>224.000</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>335.000</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>256.000</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>340.000</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>374.000</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>423.000</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>338.000</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>373.000</td>
<td></td>
</tr>
</tbody>
</table>

Now construct a histogram, but use the **Plot - 2D Bar - Vertical** plot option (see below).
Declare BinM as your x-variable and Prob as your y-variable (see below). Don’t worry about the Err or High Low Error options. Add data and OK.

You should get the following plot. Note the similarity of the probability distribution to the frequency histogram.
Today's Assignment -

**Before you leave class.**
1) generate a histogram of pebble masses
2) place your name in a text box on the plot
3) Write down the average and standard deviation of the pebble masses on your plot
4) print out the histogram of pebble masses
5) turn it in before leaving

Finish reading Chapter 7 by next Tuesday.

Make sure you have a basic understanding of the normal probability distribution.

What is the probability that a given observation will have a value that lies between ± 1 standard deviation from the mean or average value of the sample?

What is the probability that a given observation will have a value that lies more than ± 1 standard deviation from the mean or average value of the sample?

What does z represent?

What does the area from one value to another of the normal probability function represent? See Figures 7.4 and 7.5 and Table 7.5. Make sure you understand the concepts being discussed in association with the normal probability distribution.
The data used in this problem are taken from an exercise prepared by Stephen Nelson, Tulane University, for his Geology 204, Natural Disasters class (see http://www.tulane.edu/~sanelson/geol204/hmwk5_00.htm). You can visit Dr. Nelson’s site for more details on the problem. In this problem we will analyze maximum discharge data from Dry Creek – a stream in Louisiana – to estimate flood frequency and the annual probability of occurrence for floods of a given discharge. Our interest here is to introduce you to the basic idea and also to give you some additional experience using PsiPot and Excel to manipulate, plot, and analyze data.

The data are contained in an Excel file (Discharge.xls) placed on the class H:\Drive.

In this exercise you will learn how to use Excel and PsiPlot to:

1) rank a list of data (peak discharge);
2) compute the recurrence interval for observed discharges;
3) generate a log-normal plot of the recurrence intervals vs. discharge;
4) predict recurrence intervals for discharges larger than those observed in the time period covered by the data (50 years and 100 years);
5) and finally calculate the exceedence probability.

1) Compute the rank.

**EXCEL**

Using *Excel >> Help* we find that Excel has a spreadsheet function (RANK) that will allow us to rank a column or specified range of data in a worksheet.

```
| **Rank** - Returns the rank of a number in a list of numbers. The rank of a number is its size relative to other values in a list. (If you were to sort the list, the rank of the number would be its position.) |
| **Excel Syntax** |
| RANK(number,ref,order) |
| **Number** - is the number whose rank you want to find. |
| **Ref** - is an array of, or a reference to, a list of numbers. Nonnumeric values in ref are ignored. |
| **Order** - is a number specifying how to rank number. |
| • If order is 0 (zero) or omitted, Microsoft Excel ranks number as if ref were a list sorted in descending order. |
| • If order is any nonzero value, Microsoft Excel ranks number as if ref were a list sorted in ascending order. |

RANK gives duplicate numbers the same rank. However, the presence of duplicate numbers affects the ranks of subsequent numbers. For example, in a list of integers, if the number 10 appears twice and has a rank of 5, then 11 would have a rank of 7 (no number would have a rank of 6).

**Examples** |
| If A1:A5 contain the numbers 7, 3.5, 3.5, 1, and 2, respectively, then: |
| RANK(A2,A1:A5,1) equals 3 |
| RANK(A1,A1:A5,1) equals 5 |
```
**Excel Calculation**

In cell D5 of discharge.xls, type in the following =RANK(C5,C$5:C$25,0). In this Excel formula C5 is the number whose rank we wish to determine. C$5:C$25, specifies the range of numbers relative to which the value C5 is to be ranked. The dollar sign before each row number indicates that the row number will remain unchanged when the formula is copied (we’ll talk more about this in a later exercise). Copy this formula into cells D6 through D25. Examine the formula in some of the copied cells and note that the references to rows 5 through 25 remains fixed. The last term in the RANK spreadsheet function is 0, and this tells Excel to rank the numbers in descending order – that is, the largest number gets the smallest rank. Column D now contains the rank for each value of discharge in descending order.

**PsiPlot Calculation**

Copy Discharge.pdw from the H:\Drive to your G:\Drive. Open the file and note that the time is in decimal years with the beginning time set to 0 years. Make a copy of the discharge rates in column3 (C3). Then with the cursor in this column (column3) click Column >> Rank and select the descending option. Discharge rank will now appear ion column 3. Rename column 3 Rank.

2) The recurrence interval is calculated using the following formula:

\[ R = \frac{n + 1}{m}, \]

where R is the recurrence interval; n, is the number of years spanned by the data (n=20), and m is the rank of each discharge value in the data set.

**Excel Calculation:** Thus to calculate the recurrence intervals R associated with the discharge listed in cell C5, enter the formula =21/D5, Enter and copy this formula into cells E6 through E25.

Look at the numbers and note that the smallest discharge (950m³/sec) has the highest recurrence interval ~1.1. This is what we would expect: i.e. a flow rate greater than or equal to the minimum flow rate is likely to occur every year.

**PsiPlot Calculation:**

In PsiPlot use the Math>>Transform>>One Line equation option
Enter the equation R=21/Rank. Discharge rank will appear in column 4.

3) Generate a log normal plot of the recurrence interval versus discharge. Remember that your dependant variable is the discharge and the independent variable is the recurrence interval.
**Excel Plot**
To get the log normal plot in Excel, double click the x-axis to bring up the Format Axis window. Check the logarithmic scale check box. The range should be 1 to 100. Double click the discharge axis and set the min/max values to 0 and 5000.

**PsiPlot**
To get the log normal plot in PsiPlot double click on the x axis to get the x-axis format window. Under the Axis Mode drop down list select the log (decimal) display option and under Range, set the minimum to 1 (the max will be 100). On the Discharge axis set the min to 0 and max to 5000. Leave the Axis Mode set on Linear Norm.

4) Predict discharge for events with 50 and 100 year recurrence intervals.

In Excel, solve graphically by visually fitting a “best fit” line through the data points. Extrapolate this line out to 100 years and read off the discharge at 50 and 100 year recurrence intervals.

**PsiPlot Computation**
In PsiPlot we will use the Math>>Fitting>>User Defined approach.
Set the parameters in the User Defined Fitting window as follows

- **[INDVAR]**: R
- **[DEPVAR]**: Discharge
- **[PARAMS]**: A, B
- **[EQUATIONS]**:
  \[\text{Discharge} = A \times \log_{10}(R) + B\]

- **[INIT PARAMS]**:
  \[A = 1000\]
  \[B = 0.1\]

**ENDMODEL**

Click on the Compile button. You should get a “compile successful!” message box (click OK). The click Solve. Take the default Save Data Options (click OK).

The User Defined Fitting Report Window will come up. Note the correlation coefficient (\textit{corr}) under the Goodness of Fit Statistics. This should be pretty high – about 0.99). Click OK to exit.

Your spreadsheet will now contain three additional columns. One column will contain the slope and intercept.
Now given the slope and intercept, compute the peak discharge for 50 and 100 year events (i.e. floods with 50 and 100 year occurrence intervals).

5) Finally, compute the exceedence probability. The exceedence probability ($P_e$) is simply the reciprocal of the recurrence interval. $P_e=1/R$. If the recurrence interval is 5 years, for example, it’s probability of occurrence in any given year is 1/5 or 20%.

Nelson notes that flood stage occurs when discharge reaches 2000 m$^3$/sec. What is the occurrence interval and exceedence probability for a flood with this discharge?

Nelson asks us to consider the following:

Someone has offered to sell you a 4-bedroom 2-story house with a 2 car garage and swimming pool on a 1 acre lot on a relatively flat piece of ground on the banks of Dry Creek for what seems like a reasonable price of $50,000. The last time the house was flooded it cost $40,000 to repair the flood damage. How often has the house been flooded in the last 20 years? What is the probability that the house will be flooded in the first year that you own it? Would you still consider buying the house? Why or why not?

If you decide to do this for extra credit it will be due ……..
Generating the Gaussian or Normal Probability Density Function using EXCEL

The instructions below will take you through the generation and plotting of the normal probability density function:

\[ p(x) = \frac{1}{\sqrt{2\pi} \hat{s}^2} e^{-\frac{(x-m)^2}{2\hat{s}^2}} \]

using EXCEL. The probability density function provides a graphical display of the probability of occurrence of individual values in the sample of pebble mass assuming it is normally distributed with mean \( \hat{m} \) and standard deviation \( \hat{s} \).

Begin by **OPENing EXCEL** and, in column I, add the values 100 and 110 in cells I2 and I3. Click on Cell I2 and while holding down the SHIFT key click on the next cell - cell I3. Both cells will be highlighted. Drag the lower right corner of the selected cells down to row 52 or until the scrolling text box reads 600. Let up and the column fills with values from 100 to 600 at 10gram intervals.

Next go over to Column A and in cell number 5 type in mean pebble mass. In cell A6 type in standard deviation of pebble masses.

In cell B5 enter the value 350.18 and in cell B7 enter the value 48 (i.e. the mean and standard deviation of the pebble masses.)
The values in column B and rows 5 and 6 can be referred to later in our computation of \( p(m) \).

Cell references in EXCEL can be either \textbf{absolute} or \textbf{relative}. The absolute reference is useful, because it allows us to refer back to a specific cell and avoids having that value change when a formula cell into which it is entered is copied. The absolute reference is fixed reference. Whether a reference is fixed or absolute depends only on how you refer to it. The following table illustrates various absolute references.

<table>
<thead>
<tr>
<th>Formula</th>
<th>Result when formula is copied</th>
</tr>
</thead>
<tbody>
<tr>
<td>=2*$A1</td>
<td>The column remains constant – fixed</td>
</tr>
<tr>
<td>=2*A$1</td>
<td>The row remains constant – fixed</td>
</tr>
<tr>
<td>=2*$A$1</td>
<td>The row and column remain constant – fixed</td>
</tr>
</tbody>
</table>

Now in our example, the mean mass is stored in cell B5 and the standard deviation in cell B6.

\[
p(m) = \frac{1}{\sqrt{2\pi s^2}} e^{-(m-\bar{m})^2 / 2s^2}
\]

When we go to compute the normal probability density of a mass distribution having average value 350.18 and standard deviation 48 then we can make absolute reference to cells B5 and B6.

**Simplifying Complex Calculations:**

Here are some suggestions that might be useful when undertaking complex computations of the sort that you might be asked to do on the job. This advice applies whether you use PSIPlot or EXCEL.

We can break any complicated mathematical expression down into parts that can be solved for individually and then combined later in a complicated mathematical expression. The computation of \( p(m) \) is actually not that complex and you might prefer not to take this approach in the future, but the suggestions are made for you to consider. When complicated expressions are broken down into parts, individual parts can be checked for accuracy, and there is less likelihood of error showing up in the final solution.

In the present application, we could compute the factor \( \frac{1}{\sqrt{2\pi s^2}} \) separately and store it in a cell for later use. To do this, go to cell B7 and type in \( =1/(2*3.141593*SB6^2)^0.5 \). In cell A7 enter \( = \text{Inverse of square root of } 2*\pi*s^2 \). This will identify the value 0.0083 that appears in cell B7.
We could also calculate out the variable \( z = \frac{m - \hat{m}}{s} \)

Go to cell J2 and enter \( = (I2 - $B$5)/$B$6 \). This is the \( z \) value for a mass of 100 grams. You should get \( z = -5.21 \) returned to cell J1. You may be saying to yourself that you could just as easily have entered \( = (I2 - 350.18)/48 \). But again, the purpose of using the absolute cell reference in this case is to give you a simple example of its use. Absolute references can help you organize more complicated calculations than the one we just dealt with. The present example is offered as an illustration of the possible use. Copy this cell into cells J3 through J52.

**Computing the probability distribution \( p(m) \)**

Go to column K and enter the following formula in cell K2 -

\[ =B7*EXP(-0.5*J2^2) \]

\( B7 \) is the constant \( \frac{1}{\sqrt{2\pi s^2}} \)

\( J2 \) is \( z \) for a pebble having mass 200 grams. The \( J \)'s equal \( \frac{m - \hat{m}}{s} \)

\( EXP \) is the natural base \( e \).

Copy cell K2 into cells K3 through K52.

Your window should look like the one below.

**Plotting the Probability Distribution \( p(m) \) vs. \( m \)**

Before generating a plot of the probability versus mass select these two columns. We'll review the procedures for selecting non-adjacent columns. *Take Notes!*
Click on the chart wizard icon on the EXCEL menu bar
(see illustration at right)

This will bring up the Chart Wizard window (below). Select the illustrated options, then click on Next.

The next window to appear requests information about the series of cells that you want to plot. Click on the Series folder (see below).
If you wanted to, you could click on the series tab and replace the I’s with J’s to get the plot of probability versus Z than Mass. However, we have a specific interest in evaluating the probability of occurrence of various pebble masses.

Make Some Notes

Click OK
Change the plot title and axis labels to suit the problem (see below).

The Chart Location window appears. Take the default, which will place the plot in your worksheet.
Your worksheet should look something like that shown below.

Changing Absolute Cell Values

One very powerful aspect of the absolute cell referencing features in EXCEL is that this will allow you to reproduce the entire series of computations for a different mean and different standard deviation simply by changing the values you entered in cells B5 and B6.

Go ahead and change the mean value to 400. Note what happens to your plot. The change is automatically made to your plot. Now change the standard deviation to 100.

Note these various changes - Experiment
Plot Esthetics:

For a professional presentation, you may wish to enhance the appearance of your plot with different backgrounds, text colors, etc. Several cosmetic features will be illustrated in the class presentation, so please take notes!

You will find it useful to add the **drawing tools**. To do this, right click on blank areas in the upper menu panel. The menu shown below will drop down; **Click on Drawing**.

The following menu bar will appear at the bottom of your Excel page.

Click here to create a text box.
• Add Text Box

• Format text in text box

• Format background in text box

• Format Cells

• Format Chart
Your completed display might end up looking like that below.

<table>
<thead>
<tr>
<th>Mass (grams)</th>
<th>Z</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>-5.205533</td>
<td>1.06795688</td>
</tr>
<tr>
<td>110</td>
<td>-5</td>
<td>0.99912509</td>
</tr>
<tr>
<td>120</td>
<td>-4.799467</td>
<td>0.99943659</td>
</tr>
<tr>
<td>130</td>
<td>-4.560533</td>
<td>0.98583249</td>
</tr>
<tr>
<td>140</td>
<td>-4.375</td>
<td>0.97023207</td>
</tr>
<tr>
<td>150</td>
<td>-4.169687</td>
<td>0.95308886</td>
</tr>
<tr>
<td>160</td>
<td>-3.953333</td>
<td>0.93276502</td>
</tr>
<tr>
<td>170</td>
<td>-3.75</td>
<td>0.90886608</td>
</tr>
<tr>
<td>180</td>
<td>-3.541867</td>
<td>0.88241685</td>
</tr>
<tr>
<td>190</td>
<td>-3.333333</td>
<td>0.85395658</td>
</tr>
<tr>
<td>200</td>
<td>-3.125</td>
<td>0.82327404</td>
</tr>
<tr>
<td>210</td>
<td>-2.919867</td>
<td>0.78981173</td>
</tr>
<tr>
<td>220</td>
<td>-2.700333</td>
<td>0.75392319</td>
</tr>
<tr>
<td>230</td>
<td>-2.48</td>
<td>0.71465906</td>
</tr>
<tr>
<td>240</td>
<td>-2.261867</td>
<td>0.67241064</td>
</tr>
<tr>
<td>250</td>
<td>-2.033333</td>
<td>0.62703073</td>
</tr>
<tr>
<td>260</td>
<td>-1.81</td>
<td>0.57843411</td>
</tr>
<tr>
<td>270</td>
<td>-1.586687</td>
<td>0.52672348</td>
</tr>
<tr>
<td>280</td>
<td>-1.366667</td>
<td>0.47187543</td>
</tr>
<tr>
<td>290</td>
<td>-1.125</td>
<td>0.41410154</td>
</tr>
<tr>
<td>300</td>
<td>-0.894187</td>
<td>0.35432956</td>
</tr>
<tr>
<td>310</td>
<td>-0.663333</td>
<td>0.29174655</td>
</tr>
<tr>
<td>320</td>
<td>-0.4325</td>
<td>0.22650192</td>
</tr>
<tr>
<td>330</td>
<td>-0.199867</td>
<td>0.16032819</td>
</tr>
<tr>
<td>340</td>
<td>0.033333</td>
<td>0.09345961</td>
</tr>
<tr>
<td>350</td>
<td>0.288333</td>
<td>0.02813405</td>
</tr>
<tr>
<td>360</td>
<td>0.443333</td>
<td>0.00762193</td>
</tr>
<tr>
<td>370</td>
<td>0.60</td>
<td>0.00183642</td>
</tr>
<tr>
<td>380</td>
<td>0.755556</td>
<td>9E-05</td>
</tr>
<tr>
<td>390</td>
<td>0.911111</td>
<td>0E-05</td>
</tr>
<tr>
<td>400</td>
<td>1.06875</td>
<td>0E-05</td>
</tr>
</tbody>
</table>

**Statement of problem:** Compute and plot the normal probability densities of pebble masses tabulated in Table 7.1 of Waltham's text.
Generating Histograms with Excel

Up to this point we have concentrated on the use of PsiPlot to do spreadsheet calculations and basic plotting and analysis of data. This is largely because PsiPlot is easier to use in some respects than Excel, particularly for plotting, plot formatting and regression computation and plotting. The following exercise is provided to increase your familiarity with Excel functionality. Excel will generate data histograms similar to those we developed using PsiPlot.

First click on tools and examine the drop down window (see below).

Look for Data Analysis. If you can’t find it in the list, you will have to install it. To install the Data Analysis Tool Pak click on Add-Ins and then check the Analysis ToolPak option box (see below). Click OK and the add in should take place.
Now return to the Tools menu item and you should now see a Data Analysis option in the list. Select the Data Analysis option.

When you click on Data Analysis in the above list, the following window will open up. Highlight the Histogram Analysis Tool and click OK.
In the following menu, place your cursor in the Input Range box and then with your mouse left-click on the pebble mass data values and slide down selecting the data in that column. When you finish, your screen should look like that shown below.

![Image of Excel histogram dialog box]

Click on the BinRange box and select the bin cells provided for you in the worksheet. Finally, click on the Output Range box and then with the mouse sweep through a few rows and columns to give it some cells to output to. Your window should look something like the following.

![Image of Excel histogram output settings]

Also check the Chart Output option box. Then click OK. Your worksheet should look like the following.
Note that the columns of Bin and Frequency appear along with the histogram to the right.

You can edit your histogram plot to increase plot size, change label names, etc.
Left-clicking on the histogram bars will bring up the following window. From there, you can select the options folder and adjust the Gap width in the histogram to eliminate the gaps altogether (set Gap width to 0).

**Today’s assignment:**

Hand in a plot of the probability density function generated in the first part of this exercise for the pebble mass sample.

Make sure it is correctly labeled and that your name is on it.
Waltham provides us with some strike and dip data in Table 7.9. Histograms of these data along with the mean, standard deviation and standard error of individual samples are shown in the graphs below. These data are also available on my shared directory in geomath/statistics as file `strikedip.xls`. Pick up this file at the beginning of class.

What do you think? Do the distributions of strike and dip at locations A and B look different? Do you think they are statistically different?

Open EXCEL
Open File StrikeDip.xls
(you should have 4 columns of data in your file)

Before we undertake the statistical analysis of these data let’s peak under EXCEL’s hood for a second.

**Built in Functions**
Click Help
Click Microsoft Excel Help
Type "Functions" in the “What would you like to do?” box
Click Search

What you get next seems to vary but perhaps the response will look like that below.

Click on the "Worksheet Functions listed by Category"

This should yield the window below (appearance may vary depending on the version of excel and the operating system, i.e. Windows 2000 vs. Windows XP).
Depending on the version of Excel on your machine you will probably see references to Statistical Functions. Also note the variety of additional function topics that are available for your use.

Click on Statistical Functions to get a window like that at right...
Scroll down through the list and take a quick look at the various functions available for statistical analysis.

- Note the **AVERAGE** function (second from the top) in the list of EXCEL functions.

  **Example calculation:**
  Go to cell A22 and
  Type in  =average(a2:a21)

  Enter-
  The value returned to that cell is 43.15

- Scroll down the functions list and note the function **STDEV**. This function computes the standard deviation. Click on it to read about it.

  **Example calculation:**
  Go to cell A23 and enter
  STDEV(a2:a21)
  Enter -
  The value returned will be ~12.7

- Note that just above **STDEV** is the function **STANDARDIZE**. **STANDARDIZE** will return the standard normal form (z) of the variable (in this case strike or dip).

  **Example calculation:**
  Go to cell E2 and enter
  =STANDARDIZE(A2,$A$22,$A$23)
  The three parameters in parentheses refer to the value you wish to calculate z for (A2 in this case), the **mean** (in cell A22) and the **standard deviation** (in cell A23).

  *Remember that the $A$22 type references are absolute references. No matter where the formula is copied, the reference remains unchanged.

  ENTER -
  The returned value should be ~0.62
Next copy cell E2 into cells E3 through E21. This yields the standard normal form (z) for each value of A (Strike at location A) in cells A2 through A21. Because absolute references are made to cells containing the mean and standard deviation these values did not change when copied.

- Scroll back up the help window opened earlier and note the function NORMDIST. NORMDIST returns the normal probability for a given sample value.

**Example Calculation:**
The variables used to define the NORMDIST operation are individual values of x (e.g. A2, A3, etc.), the mean (e.g. $A22$), the standard deviation (e.g. $A23$), and the last variable tells the function whether to compute the cumulative probability or the individual probabilities.

Go to cell F2 and Enter -
=normdist(a2,$A$22,$A$23,FALSE)
*FALSE indicates that individual probabilities will be calculated.
ENTER -
The value 0.026 will be returned.
Now copy cell F2 into cells F3 through F21.

**Graph the normal density distribution as derived from the mean and standard deviation of measured strikes listed in column A.**
Click on cell A2, hold down the left mouse button and drag down to cell A21. Then sort in ascending order (use the A-to-Z button). Note that not only have the A-values been sorted but the cells F2 through F21 have been shifted and retain their original association with individual cells in column A.

Left Click on cell A2, hold down the left mouse button and drag down to cell A21.
Cells A2 through A21 will be highlighted.

Next hold the Ctrl key down, left click on cell F2, and drag down to cell F21. Cells A2 through A21 and F2 through F21 should all be highlighted.
Click on the Chart Wizard icon.
Select XY-Scatter
then Next
(X and Y series should be correctly defined.)
Click Next

Then Add Axis Labels
Your plot should look similar to that at right.

FINISH
We will spend a few minutes experimenting with fonts, and then we'll return to the main objective of today's lab exercise, which is to evaluate the statistical significance of differences in strike and dip between locations A and B using the t-test.
THE T-TEST
- Evaluate the significance of differences in mean using the t-test.

In your help topics list (see right), note the function TTEST.

The TTEST function returns the probability that the two averages could actually be representative of the same parent population; i.e.

Click on TTEST to read about its use (see above).

In cell G24 type in - Probability that average strike at locations A and B is the same.

Click and pull the left edge of cell G24 to the right so that the entire statement fits in this cell.

Format cells G24 and H24 (follow in class)
In cell H24
Enter =TTEST(A2:A21,C2:C21,1,2)

In cell G25 type in - Probability that average dip at locations A and B is the same.
It should not be necessary to make any adjustments to the width of cell G25.

Format cells G25 and H25 (as above)
Then in cell H25
Enter =TTEST(B2:B21,D2:D21,1,2)
T-Tests the old-fashioned way

It would also be a good idea to learn how to do the t-test using t-statistics tables. The table below lists critical values of t (remember this is just like z – the number of standard deviations from the mean – accept that it has been adjusted for sample size) corresponding to one-tailed probabilities (expressed as percentages) of 10, 5, 2.5, etc.

<table>
<thead>
<tr>
<th>Number of Degrees of Freedom, ( \nu )</th>
<th>Significance Level, ( \alpha ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>3.078</td>
</tr>
<tr>
<td>2</td>
<td>1.886</td>
</tr>
<tr>
<td>3</td>
<td>1.638</td>
</tr>
<tr>
<td>4</td>
<td>1.533</td>
</tr>
<tr>
<td>5</td>
<td>1.476</td>
</tr>
<tr>
<td>6</td>
<td>1.440</td>
</tr>
<tr>
<td>7</td>
<td>1.415</td>
</tr>
<tr>
<td>8</td>
<td>1.397</td>
</tr>
<tr>
<td>9</td>
<td>1.383</td>
</tr>
<tr>
<td>10</td>
<td>1.372</td>
</tr>
<tr>
<td>11</td>
<td>1.363</td>
</tr>
<tr>
<td>12</td>
<td>1.356</td>
</tr>
<tr>
<td>13</td>
<td>1.350</td>
</tr>
<tr>
<td>14</td>
<td>1.345</td>
</tr>
<tr>
<td>15</td>
<td>1.341</td>
</tr>
<tr>
<td>16</td>
<td>1.337</td>
</tr>
<tr>
<td>17</td>
<td>1.333</td>
</tr>
<tr>
<td>18</td>
<td>1.330</td>
</tr>
<tr>
<td>19</td>
<td>1.328</td>
</tr>
<tr>
<td>20</td>
<td>1.325</td>
</tr>
<tr>
<td>21</td>
<td>1.323</td>
</tr>
<tr>
<td>22</td>
<td>1.321</td>
</tr>
<tr>
<td>23</td>
<td>1.319</td>
</tr>
<tr>
<td>24</td>
<td>1.318</td>
</tr>
<tr>
<td>25</td>
<td>1.316</td>
</tr>
<tr>
<td>26</td>
<td>1.315</td>
</tr>
<tr>
<td>27</td>
<td>1.314</td>
</tr>
<tr>
<td>28</td>
<td>1.313</td>
</tr>
<tr>
<td>29</td>
<td>1.311</td>
</tr>
<tr>
<td>30</td>
<td>1.310</td>
</tr>
<tr>
<td>40</td>
<td>1.303</td>
</tr>
<tr>
<td>60</td>
<td>1.296</td>
</tr>
<tr>
<td>120</td>
<td>1.289</td>
</tr>
<tr>
<td>( \infty )</td>
<td>1.282</td>
</tr>
</tbody>
</table>

Source: From Table 21, The Penguin-Honeywell Book of Tables, copyright F. W. Kelway (ed.) and Honeywell Controls Ltd. (E.D.P. Division), 1968.
When you do it by hand you actually have to think about what you are doing and have a little better understanding of what's going on. In the preceding example, we undertook the t-test using the expression =TTEST(A2:A21,C2:C21,1,2). A2:A21 and C2:C21 define the series of numbers we wish to compare. This is a two-sample t-test as distinguished from the one-sample t-test in which you attempt to assess whether a particular mean is different from what you think the overall population mean is.

The two-sample t-test evaluates the statistical significance of the difference between the means of two samples. Our test statistic has the form

\[ t = \frac{\overline{D}_1 - \overline{D}_2}{s_e} \]  

(doesn’t this remind you of z?)

where \( s_e = s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \) and \( s_p \) is the pooled estimate of the standard deviation, found by combining the sample variances of the two data sets as follows -

\[ s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} \]

In this formula \( s_1 \) and \( s_2 \) are the unbiased estimates of the standard deviation and are equivalent to values returned using the STDEV excel function or in the summary statistics reported by PsiPlot. You could return to your EXCEL spreadsheet and make this computation. If you do, you can verify that the pooled variance is 115.98, that the pooled estimate of the standard deviation is 3.4 and that the t-statistic - characterizing the difference between the two means in terms of multiples of the pooled estimate of the standard deviation - is 5.2.

Thus these two means differ by 5.2 times the pooled estimate of the standard deviation. Remember a z-statistic of 1.96 would be significant at the 95% confidence level (5% - two tailed or 2.5% - one tailed). So we suspect that such a high t-statistic indicates a significant difference between the two means.

We can consult the t-tables for specific levels of significance. Note that the degrees of freedom in this case is \( n_1 + n_2 - 2 \) or 38. The table doesn't have a listing for 38 degrees of freedom, but 40 is close. The critical values of t are for the one-tailed probabilities. For example, for degrees of freedom \( N=\infty \), 10% significance is met by t-values greater than or equal to 1.282. Refer to Waltham’s Table 7.6 and notice that 80% of the area under the normal probability curve lies approximately between \( \pm 1.3 \) standard deviations from the mean. At \( \infty \), \( t = z \). With 80% of the area between \( \pm 1.3 \) standard deviations, 10% lies in either of the two tails.

Looking out at the rightmost column, \( \alpha \) of 0.1 implies a one-tailed probability of 1 in one thousand that \( t \) could be greater than or equal to 3.307. Our value of 5.2 is larger than that but notice we cannot assign the exact probability to our value of \( t \). We can only say the the probability for this value of \( t \) is less than one in a thousand. EXCEL does provide the
exact probability but the tables are usually not detailed enough for you to do that. However, to be able to say that there is less than 1 chance in 1000 that one of these means is greater than the other is good enough.

**Conducting the t-test using PsiPlot**

If you want to try this out, you can copy the columns of strike and dip data from EXCEL into PsiPlot, Your spreadsheet will look something like that shown below.

<table>
<thead>
<tr>
<th>StrikeA</th>
<th>DipA</th>
<th>StrikeB</th>
<th>DipB</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>51.0000</td>
<td>22.0000</td>
<td>60.0000</td>
</tr>
<tr>
<td>2</td>
<td>45.0000</td>
<td>23.0000</td>
<td>54.0000</td>
</tr>
<tr>
<td>3</td>
<td>44.0000</td>
<td>21.0000</td>
<td>69.0000</td>
</tr>
<tr>
<td>4</td>
<td>61.0000</td>
<td>24.0000</td>
<td>59.0000</td>
</tr>
<tr>
<td>5</td>
<td>37.0000</td>
<td>21.0000</td>
<td>58.0000</td>
</tr>
<tr>
<td>6</td>
<td>11.0000</td>
<td>23.0000</td>
<td>59.0000</td>
</tr>
<tr>
<td>7</td>
<td>26.0000</td>
<td>23.0000</td>
<td>66.0000</td>
</tr>
<tr>
<td>8</td>
<td>54.0000</td>
<td>21.0000</td>
<td>62.0000</td>
</tr>
<tr>
<td>9</td>
<td>42.0000</td>
<td>22.0000</td>
<td>48.0000</td>
</tr>
<tr>
<td>10</td>
<td>29.0000</td>
<td>22.0000</td>
<td>62.0000</td>
</tr>
<tr>
<td>11</td>
<td>48.0000</td>
<td>22.0000</td>
<td>53.0000</td>
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<td>34.0000</td>
<td>18.0000</td>
<td>72.0000</td>
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<td>13</td>
<td>50.0000</td>
<td>26.0000</td>
<td>62.0000</td>
</tr>
<tr>
<td>14</td>
<td>61.0000</td>
<td>24.0000</td>
<td>69.0000</td>
</tr>
<tr>
<td>15</td>
<td>36.0000</td>
<td>26.0000</td>
<td>70.0000</td>
</tr>
<tr>
<td>16</td>
<td>47.0000</td>
<td>30.0000</td>
<td>41.0000</td>
</tr>
<tr>
<td>17</td>
<td>30.0000</td>
<td>21.0000</td>
<td>59.0000</td>
</tr>
<tr>
<td>18</td>
<td>54.0000</td>
<td>25.0000</td>
<td>76.0000</td>
</tr>
<tr>
<td>19</td>
<td>48.0000</td>
<td>21.0000</td>
<td>54.0000</td>
</tr>
<tr>
<td>20</td>
<td>55.0000</td>
<td>21.0000</td>
<td>64.0000</td>
</tr>
<tr>
<td>21</td>
<td>29.0000</td>
<td>22.0000</td>
<td>62.0000</td>
</tr>
</tbody>
</table>

To conduct the t-test click on Data and select **t-test** as shown below.

![PsiPlot Data menu](image-url)
In the t-test window, select the two datasets you want to compare then click OK.

The following summary window appears -

That's all there is to it. Note that the t-values obtained by 
PsiPlot and EXCEL are in agreement, and also note that 
PsiPlot explicitly states the 
probability of such an 
ocurrence.
For Today

Print off the Excel spreadsheet we've been working on in today’s lab. Hand in with your name on it.

Homework Assignment

Do problem 7.13 on page 134. Evaluate significance in the differences of mean using both the EXCEL t-test function and the "old fashioned" approaches discussed above (direct calculation of the pooled variance and standard error, and the use of t-statistic tables to evaluate alpha level).

1. Report your work in standard form including a statement of the problem and a summary of results.
2. Show the computation of the pooled variance and standard error.
3. Present histograms of the Mount Monger and Emu data sets.
4. On these histograms note the mean, standard deviation and standard error of each individual sample.

Due ______________
The derivative is essentially the slope of a function at a given point. We’ve talked about slopes on numerous occasions throughout the semester. We estimated age-depth relationships and sedimentation rates for different periods of time. In most of the cases we’ve worked with up to this point, those slopes have been constant over a certain time period. Sedimentation rates in the North Sea for example were roughly linear during the 0 to 10,000 period before present and the 10,000 to 15,000 year before present periods. We noted that in many instances, we would expect the age-depth, depth-porosity relationships to vary with depth because of the influence of compaction. As time passes, the increased weight of sediment bearing down on a layer deposited at an earlier time will push grains closer together and reduce the open space (or pore space) within the layer. The layer will get thinner and the porosity will decrease with increased depth of burial.

The porosity depth relationship in unconsolidated sediments is often written as

$$\phi = \phi_0 e^{-cz}.$$  

In this equation, $\phi_0$ is the initial porosity at a depth $z = 0$, and $c$ is a constant (a compaction factor). The porosity depth relationship is shown below over the 1 to 5 km range of depths ($z$) for an initial porosity of 0.5 and compaction factor of 0.5 km$^{-1}$. The rate of change of porosity between 1 and 2 kilometers could be estimated as $\frac{\Delta \phi}{\Delta z}$ where $\Delta \phi$ is the difference in porosities measured at 1 and 2 kilometers, $\phi_2 - \phi_1$, and $\Delta z$ is the difference in the corresponding depths $z_2 - z_1$.

$\phi_2$, & $\phi_1$ are 0.184 and 0.304 at depths ($z$) of 2 and 1 km, respectively. This gives a slope or gradient of -0.12 per kilometer for $\frac{\Delta \phi}{\Delta z}$. Spatial variations are usually referred to as gradients.
while temporal changes are referred to as *rates*. You can see from the graph that this \( \frac{\Delta \phi}{\Delta z} \) is an average estimate of the porosity/depth gradient between 1 and 2 kilometers since the line representing this gradient (or slope) intersects the line at these two points. This measure provides a useful general reference to how porosity is changing between depths of 1 and 2 kilometers in this area, but does not tell us how the porosity changes at any one point in particular.

We could ask a similar question, but refine our depth range down to 100 meters instead of 1 kilometer. For example, in the plot below, we compare the gradient between 1 and 1.1 km to the gradient between 1 to 2 km. \( \frac{\Delta \phi}{\Delta z} \) in this case is \( \frac{0.289 - 0.303}{0.1} \), or \( -0.14 \) per kilometer. The gradient is a little steeper in the shallower part of the curve.

As Waltham notes, the derivative is a measure of the slope of a curve in the limit that the \( \Delta z \) (or whatever the independent variable happens to be) goes to zero. As \( \Delta z \) goes to zero, we are looking at the slope of a line that intersects the curve at only one point. This makes that line a tangent. The derivative is the tangent of the line intersecting a curve at a single specified point.

In class we note that the derivative of the exponential function \( \frac{de^x}{dx} \) is just \( e^x \). The function is indestructible. It does not change with successive differentiations (i.e. \( \frac{d^2y}{dx^2} = e^x \), and so on); no matter how many times you differentiate \( e^x \), the result is the same: \( e^x \). However, if \( x \) is multiplied

---

*Porosity-Depth Relationship*

![Graph showing porosity-depth relationship with gradients labeled.]

As Waltham notes, the derivative is a measure of the slope of a curve in the limit that the \( \Delta z \) (or whatever the independent variable happens to be) goes to zero. As \( \Delta z \) goes to zero, we are looking at the slope of a line that intersects the curve at only one point. This makes that line a tangent. The derivative is the tangent of the line intersecting a curve at a single specified point.

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by a constant, the derivative does change. If we differentiate the function \( e^{-cz} \) with respect to \( z \) (i.e. \( \frac{de^{-cz}}{dz} \)), the derivative is computed in a two-step process – we have to use the chain rule.

First you evaluate the derivative of \( e^x \) with respect to \( x \), where \( x = -cz \), then we multiply that result by the derivative of \( -cz \) with respect to \( z \). Recall the general definition of the chain rule discussed in class today. To use the chain rule we evaluate the product \( \frac{de^x}{dx} \frac{dx}{dz} \). \( \frac{dx}{dz} \) is simply \( e^x \), and \( \frac{dx}{dz} = \frac{d(-cz)}{dz} = -c. \)

In this two-step process, the \( dx \)'s cancel out and we get \( \frac{de^{-cz}}{dz} \).

\( y = e^{-cz} \) is a composite function, or a function of another function. We consider the term in the exponent as a separate function: in this case, \( x(z) = -cz. \)

---

**In Class Lab Exercise**

The lab exercise is designed to help you prove to yourself, that the process of taking the derivative is nothing more than the process of computing the slope – i.e. the ratio of the differences \( \frac{\Delta y}{\Delta z} \) at any given point or at successive points. In this first example, you’ll see that the

\[ \frac{\Delta y}{\Delta z} \]  (we’re using \( z \) instead of \( x \) in this case) for \( y = \frac{de^{-cz}}{dz} = -ce^{-cz}. \)

We’ll use Excel to go through the numerical calculations of \( \frac{d\phi}{dz} \) where we assume \( \phi = e^{-z} \) for simplicity. Thus, in this case \( \frac{d\phi}{dz} = -e^{-z}. \) Let’s calculate the slopes, point-by-point, and see if that’s the way it works out.

- **Open Excel.** In cell A1 enter \( Z \), then generate a column of numbers from 0 to 10 with increments of 0.1 running from cell A2 to A102.
  1. In cell B1 add the title porosity (\( z \)). In cell B2 enter the formula \( \text{=exp(-a2)} \) and copy this formula into cells B3 through B102. In this example, we assume that \( \phi_0=1 \) (This is an impossible scenario, but our main interest here is to calculate the derivative of the natural exponential function). The result you get in cell B2 will be 1.
  2. **Title column C dPhi/dz (i.e. \( \frac{d\phi}{dz} \)).** Leave cell C2 blank and enter \( (b3-b2)/0.1 \) in cell C3.
     Copy the formula into cells C4 through C102.
  3. What is the 0.1 for? 0.1 is the \( \Delta z \). Without \( \Delta z \) in the denominator the differences in the values listed in columns B and C do not equal the negative of the porosity we expect to get in the calculation.
Don’t forget the $\Delta z = 0.1$ in the denominator. Your result will look very much like that below with the values in columns A through C shown at left plot at right.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$Z$</td>
<td>Property (+) $d\Phi/dZ$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.1</td>
<td>0.5948374</td>
<td>-0.95163</td>
</tr>
<tr>
<td>4</td>
<td>0.2</td>
<td>0.8187368</td>
<td>-0.86107</td>
</tr>
<tr>
<td>5</td>
<td>0.3</td>
<td>0.7408182</td>
<td>-0.77913</td>
</tr>
<tr>
<td>6</td>
<td>0.4</td>
<td>0.67032</td>
<td>-0.70498</td>
</tr>
<tr>
<td>7</td>
<td>0.5</td>
<td>0.6055307</td>
<td>-0.63789</td>
</tr>
<tr>
<td>8</td>
<td>0.6</td>
<td>0.5498116</td>
<td>-0.57719</td>
</tr>
<tr>
<td>9</td>
<td>0.7</td>
<td>0.4956853</td>
<td>-0.52226</td>
</tr>
<tr>
<td>10</td>
<td>0.8</td>
<td>0.449329</td>
<td>-0.47256</td>
</tr>
<tr>
<td>11</td>
<td>0.9</td>
<td>0.4056567</td>
<td>-0.42759</td>
</tr>
<tr>
<td>12</td>
<td>1</td>
<td>0.3678794</td>
<td>-0.3869</td>
</tr>
</tbody>
</table>

4. The basic definition states that the slope $\frac{\Delta \phi}{\Delta z}$ in the $\lim_{\Delta z \to 0}$ becomes $\frac{d\phi}{dz} = -e^{-z}$.

5. We have shown that the simple process of calculating successive slopes over small steps in depth ($\Delta z$) provides a very good approximation of the derivative.

6. **Prepare a plot of the result.** Plot the values in columns B and C on the same chart. Hand the plot in with your name on it before the end of class today.

The basic lessons illustrated in this exercise are 1) that $\frac{d\phi}{dz} = -e^{-z}$, and 2) computers can easily compute derivatives as long as we use a small difference in our independent variable (and don’t forget to include that $\Delta z$ in the denominator!).

Also note that you can use this approach to experiment with other simple derivatives. For example, try calculating successive differences for the sine, cosine or other favorite function.
This exercise asks you to use the understanding you have of various functions and their combinations to visualize the function and its derivative without computation. In class, we used, as an example, the function \( s = S_{\text{max}} (1 - e^{-t/\tau}) \). This function is used to describe sediment thickness accumulating in an extensional basin through time. We distribute the constant \( S_{\text{max}} \) through \((1 - e^{-t/\tau})\) to get \( S_{\text{max}} - S_{\text{max}} e^{-t/\tau} \). This is just a combination of two functions: a horizontal line with zero slope and constant value \( S_{\text{max}} \) minus the exponential decay function scaled by \( S_{\text{max}} \). Using the simple case where \( S_{\text{max}} = 1 \).

As you can see in the plot above, the operation of taking \( 1 - e^{-t/\tau} \) flips the decay function a rising function that approaches the value 1 asymptotically.

Before looking at the following page, visualize the derivative of the rising function \( 1 - e^{-t/\tau} \). In a general way, how will the slope of the tangent line vary with increasing time?

From looking at how the slope of the tangent line will vary through time, you expect your derivative will start off with a high value and then drop off approaching zero through time. The result you calculate should have this property.
As you work through problems 8.13 and 8.14 use this visualization process to crosscheck your results. As you can see below, the derivative has the properties we expected to see: it starts at a maximum and drops exponentially.
We'll go quickly through two examples today; one, which will use PsiPlot as the analytical tool, and another that employs EXCEL. We’re getting close to the final so take this as opportunity to ask questions about the operation of these two software packages.

**Erosion**


In earlier presentations we considered the influence of isostacy on variations in crustal thickness needed to compensate topographic features. In one exercise we considered the interplay between erosion and isostacy. We found, for example, that removing two kilometers of material from the mountain did not reduce the mountain height by 2km, because there was rebound of the crustal root in response to erosion.

In the present example, we visit the issue of erosion rates from a quantitative point of view. Erosion rates are considered to be proportional to relief, as expressed by the following equation -

\[
\frac{dh}{dt} = -\alpha h
\]  

(1)

In this equation, \( h \) is the relief, \( dh/dt \) is the erosion rate (change of relief with change in time) and \( \alpha \) is an erosion constant that has units of inverse length. Although we haven't gotten into integration, but you have probably already dealt with the integration of functions like this in your introductory calculus class. Integration of this equation yields a result we've seen before. Rearrange the above to obtain

\[
\int_{h_0}^{h} \frac{dh}{h} = \int_{0}^{t} -\alpha dt
\]  

(2)

and integrate - i.e.

\[
\int_{h_0}^{h} \frac{dh}{h} = \alpha \int_{0}^{t} dt
\]  

(3)

where \( h_0 \) is the initial relief, \( h \) the final relief and \( t \) the interval of time over which the erosional process is followed.
This integration yields

\[ \ln(h) - \ln(h_0) = -\alpha t \quad (4) \]
\[ \ln\left(\frac{h}{h_0}\right) = -\alpha t \quad (5) \]

Then take both sides of the above equation to be exponents of e (the natural base) yields

\[ \frac{h}{h_0} = \exp(-\alpha t) \quad (6) \]

and thus

\[ h(t) = h_0 \exp(-\alpha t) \quad (7) \]

Note that the erosion process behaves similar to porosity variations as a function of depth resulting from compaction (page 56 and exam problem), the growth rates of reefs (problem 2.15), and the radioactive decay processes (problem 2.13).

Data presented in the table below are considered to be representative of erosion rates typical of mid-latitude drainage basins. Note that denudation rate is \( dh/dt \) and relief is \( h \). Referring to equation (1), note that \( dh/dt \) is our \( y \)-variable and \( h \) is our \( x \)-variable.

<table>
<thead>
<tr>
<th>Relief (meters)</th>
<th>Denudation Rate (m/ma)</th>
</tr>
</thead>
<tbody>
<tr>
<td>199</td>
<td>55</td>
</tr>
<tr>
<td>406</td>
<td>68</td>
</tr>
<tr>
<td>804</td>
<td>127</td>
</tr>
<tr>
<td>1189</td>
<td>129</td>
</tr>
<tr>
<td>897</td>
<td>150</td>
</tr>
<tr>
<td>1384</td>
<td>160</td>
</tr>
<tr>
<td>1585</td>
<td>219</td>
</tr>
<tr>
<td>1871</td>
<td>247</td>
</tr>
<tr>
<td>2073</td>
<td>295</td>
</tr>
</tbody>
</table>

**Problem:** 1) Compute the erosion coefficient \( \alpha \), and 2) determine how many millions of years are required to reduce a mountain whose initial elevation is 2000 meters to an elevation of only 100 meters. In part 2 compute \( h(t) \) for times ranging from 0 to 70 My at 1My intervals. Plot your result and note the location of the point in time where relief has dropped to 100 meters.
Open PsiPlot and enter the above data into PsiPlot spreadsheet

Your PsiPlot spreadsheet should look like that shown at right

These data should follow the relationship defined in equation (1) i.e. \( \frac{dh}{dt} = -ah \). This is just a straight line, and the intercept in this case is 0. We need to use PsiPlot’s linear regression solver, but note that we have a slightly different problem. The general form of the linear regression relationship has both intercept and slope, but we know that the equation we wish to solve for has an intercept of 0. It is in the form: \( y = ax \).

PsiPlot handles this using “Pre-Defined” Models under Math>Fitting>Pre-defined Model (see right).
Take the Pre-defined option and the window at right will appear.

Note that **Linear 1** (first in the list) is the one we want to use.

This listing is worth coming back to for future reference and exploration.

The appropriate columns show up as the default selections for X and Y. **Just Click OK** and you’ll get the following summary window.

The fitting report lists a derived slope of 0.135. This is \( \alpha \) in the above erosion rate equation.
Part 1) is complete and for part 2) we need to generate computations of $h$ over a 70 million year time period with initial elevation of 2000 meters. We can use that data to determine the time it takes to wear this 2000 meter mountain down to 100 meters or we could compute it directly by solving for $t$ in equation 4).

This is something you should be good at by now. Just as a reminder, do the following:

**Data>Fill Selection>Algebraic** (start value is 0, end is 70 so place values in rows 1 through 71 using an increment of 1). **Label this column $t$.**

Then in the adjacent column use Math Transform and enter the equation $H=2000*\exp(-0.1354*t)$ and enter. Calculated values of $h(t)$ will appear. Your spreadsheet should look something like that at right.

Next, as requested, plot your data and note the point on the curve where $h =$100 meters as shown below.

![Relief vs. time graph](image-url)

This 2000 meter mountain is eroded to 100 meters of relief in approximately 23 My.
Finally, compute $t$ for $h = 100$ meters. As noted above, we could simply solve equation 4) $\ln(h) = \ln(h_0) - \alpha t$ for $t$, when $h = 100$. That yields $\ln(20)/\alpha$ to obtain $22.1$ My.

**Summary of results:** Erosion rate is assumed to vary in direct proportion to relief. This leads to exponential reductions of relief with time. In the present example, the height of a 2000 meter mountain is reduced by 1900 meters in 22.1 million years, however, in the following 22 million years, there is only a 95 meter drop in relief. It is interesting to note that while the erosion rate decreases rapidly with time, the percentage of remaining material eroded in unit time remains the same. For example, during the first 22.1 million years, mountain relief decreased 95% (from 2000 meters to 100 meters), and in the succeeding 22.1 million years, the mountain relief decreases again by 95% (form 100 meters to 5 meters), and so on. The same proportion of the remaining relief (95%) is removed in consecutive 22.1 My time intervals.

This is a basic characteristic of exponential decay processes whether it be accumulation rate, erosion rate, porosity variations with depth, radioactivity …etc.
Problem 8.14 (Waltham page 157-158): In this problem we examine salinity variations in a restricted basin. Salinity increases with distance from the ocean inlet following the relationship

\[ s = s_0 \alpha X / (\alpha X - x) \tag{7} \]

(see Waltham page 158).

\( s_0 \) is the salinity of the seawater at the inlet, \( \alpha \) is a constant and \( X \) is the width of the restricted basin.

Take a few moments and evaluate the derivative \( ds/dx \).

Assume \( s_0 = 30 \text{ ppm} \), and \( X = 10000 \text{ meters} \).

Compute \( S(x) \) and \( ds/dx \) for three different values of the constant \( \alpha \). \( (\alpha = 1.5, 3, \text{ and } 4.5) \)

Make calculations at 100 meter intervals across the width of the basin.

Plot \( S(x) \) for the three values of \( \alpha \) on a single graph and label. Do the same for \( ds/dx \) versus \( x \).

**Steps**

- First generate values of \( x \) from 0 to 10,000 meters at 100 meter intervals. This will give you 101 calculation points.
- Enter constants (\( s_0 \) and \( X \)) in cells for later reference.
- Program in the formula for \( s(x) = s_0 \alpha X / (\alpha X - x) \). Your EXCEL formula should (with \( \alpha = 1.5 \)) look like =$C$21*1.5*$C$20/(1.5*$C$20-$A2). \( s_0 \) has been stored in cell cell C21 and \( X \) in C20. The cell reference $A2 specifies the x-value. This reference employs mixed absolute and relative cell references. In this form, $A2 indicates that the column reference will always be column A, however, the row reference will be allowed to vary from 2 to 102 when copied.
- Program in the formula for the derivative \( ds/dx = s_0 \alpha X / (\alpha X - x)^2 \). Your EXCEL formula (with \( \alpha = 1.5 \)) will be =$C$21*1.5*$C$20/(1.5*$C$20-$A2)^2.
- Plot up your results. Select data series first before activating the Chart Wizard. Label your plots accordingly.
Salinity variations across the basin vary non-linearly. Salinity and the gradient of salinity both increase rapidly about midway across the basin. The results suggest that \( \alpha \) plays the role of a "circulation coefficient." When \( \alpha \) increases, salinity increase and rate of salinity-increase across the basin are significantly reduced. \( \alpha \) is probably related to the water depth at the inlet and the influence this has on allowing circulation of water through the basin to be maintained.

The EXCEL worksheet and plots are shown below.

---

**Questions??**

Before leaving class hand in plots of relief versus time, and the salinity gradients across the restricted basin.
We'll take a quick look at an additional example of math applications in geology – our last one for the semester! This is another geomorphology example, and deals with the evolution of landscapes through time.

**Landscape Evolution**

Along with the preceding erosion problem the following problem is also taken from Steve Sheriff's site at http://www.cs.umt.edu/GEOLOGY/classes/Comp_Geol/compgeol.htm. Also note that Charlie Angevine (University of Wyoming) suggested the ideas presented by Sheriff. The background for the computational approach to landscape evolution illustrated here comes from Slingerland, Harbaugh, and Furlong (*Simulating Clastic Sedimentary Basins* (1994, Prentice Hall, 220p)).

The erosion problem gave us an appreciation for the time frame over which landscape features will erode. In the current problem we consider how landform shape will evolve through time. The basic equation governing this process is the diffusion equation –

\[
\frac{dh}{dt} = k \frac{d^2h}{dx^2} \tag{1}
\]

where \(h\) is the elevation, \(dx\) is differential distance along the profile, \(k\) is a diffusion constant, and \(dt\) is a small span of time during which we examine the change in landform shape.

Note that there is a second derivative of elevation in the equation with respect to the spatial position on the line. First we compute the change in elevation at a given point along the profile. This is just the gradient or slope of the surface. We next compute the change in the slope with distance. That is the second derivative. So if the slope is changing rapidly – either rising toward the top of a hill or dropping into a valley, the second derivative will be large. In these areas, one will either have the greatest erosion rates with time or the least, respectively. In the latter case, we have assumed that there is no transport, and that internal valleys within the profile will fill up.

The equation represents a condition in which changes in the elevation of a point along a profile are proportional to changes in slope bordering that point and to the time span over which erosion occurs. To compute how a profile will evolve, we must first compute \(\Delta h\) for a certain time period \(\Delta t\) and then subtract or add \(\Delta h\) to the original profile elevation. So that we can compute these changes on the computer, we look at sizable, but still small changes in time space and elevation. Thus, from equation 1, our computer computation starts with
\[
\frac{\Delta h}{\Delta t} = k \frac{\Delta (\Delta h)}{\Delta x \Delta x}
\]
which becomes \[
\Delta h = k \frac{\Delta}{\Delta x} \left[ \frac{\Delta h}{\Delta x} \right] \Delta t
\]

To begin our computation we compute the inside term, \(k \left[ \frac{\Delta h}{\Delta x} \right] \), first. Let’s call this term \(G\), which is basically the gradient scaled by the diffusion constant. Then we compute the spatial change in this quantity over a certain time period: i.e.,

\[
\Delta h = \frac{\Delta G}{\Delta x} \Delta t
\]

Before we go undertake this computation on Excel, let’s look at the assumptions made by this equation. As always, the equations you use will have certain assumptions that go along with them. As Sheriff notes, in this case, the following assumptions are being made:

- We approximate a smooth topographic profile with a series of adjacent spreadsheet cells, each holding the average elevation of the surrounding area.
- We assume that the material being eroded, transported and deposited is homogeneous.
- We assume that the rivers at the edge of our landscape transport all the debris of erosion supplied to them and that they do not incise over time.
- We assume a two-dimensional landscape with no transport in and out of the plane of the cross section.
- We make no distinction between the different types of erosion, but consider only their long-term net effects.
- We assume the long-term average flux of materials on a slope is proportional to the slope.

Let’s set up the computational procedures.

As always, we have certain constants we can place in a prominent position within the worksheet and make absolute references to in our computations. The constants in the current application will be the diffusion constant \(k = 20,000\text{km}^2/\text{ma}\), \(\Delta x = 10\text{km}\), and \(\Delta t = 0.001\text{ma}\). Also insert the computation \(k*\Delta t/\Delta x^2\) into one cell and label it. After the computational procedures are all set up, you can change the values of \(\Delta x\), \(\Delta t\), and \(k\) to determine their relative effect on profile erosion, but you will want to be sure to keep the \(k*\Delta t/\Delta x^2\) term <0.5. When this term exceeds 0.5 you will see some interesting effects.

Next, we must specify our topographic profile. For the purposes of illustration, I will use the same profile that Sheriff does in his example, but you can change this around on your own, if you are interested to see how different profile shapes erode.

The starting profile has the following elevations in kilometers spaced at 1km intervals:
0.0, 0.5, 1, 2, 3, 4, 4, 3, 3, 2, 2, 1, 1.5, 1.5, 2, 1.5, 1, 0.5, 0.0
At this point, your spreadsheet may look like that shown below.

![Spreadsheet Image]

Let's look at how we can code in the computation of \( k \left[ \frac{\Delta h}{\Delta x} \right] \). In the spreadsheet you have sample distances in row 5, cells B5 through T5. The starting elevations are listed below in cells B6 through T6. To compute the term \( k \left[ \frac{\Delta h}{\Delta x} \right] \) for the elevation in cell G6 we do this by incorporating the slope to the left and right of the point. So we need actually to compute two terms: \( \frac{\Delta h}{\Delta x}_{\text{LEFT}} \) and \( \frac{\Delta h}{\Delta x}_{\text{RIGHT}} \). Since erosion generally reduces the elevation these terms are negative, and we need to tell Excel to compute

\[
\Delta G = \left( -\left( \frac{k}{\Delta x} \right) \Delta h_{\text{LEFT}} - \left( \frac{k}{\Delta x} \right) \Delta h_{\text{RIGHT}} \right),
\]

hence the Excel formula looks like:

\[((-B$3/B$1)*(G6-F6))+(-($B$3/$B$1)*(G6-H6)).\]

We complete the computation by carrying out the calculation shown in equation 2:
\[ \Delta h = \frac{\Delta G}{\Delta x} \Delta t . \]

The completed Excel formula looks like this:

\[ = \text{G6} + (-(\text{B$3/$B$1}) \times (\text{G6} - \text{F6})) + ((\text{B$3/$B$1}) \times (\text{G6} - \text{H6})) / \text{B$1}) \times \text{B$2}. \]

(Don’t confuse G in the Excel formula with \( G \), the gradient.) \( \Delta x \) is in cell \$B$1, and \( \Delta t \) is in cell \$B$2. Recall that what we have computed is the erosion of material out of the section, hence we add the result to the elevation in the previous step.

**Today’s Class Exercise**

- Cut the highlighted green cells in row 7 and paste in the next 70 to 80 lines below.
- Select the distances along the profile in row 6, the initial topographic relief in row 7 (remember to hold down on the Ctrl key when you are doing this), then select every 10th row down to the end of your data set (e.g. rows 16, 26, 36, …., etc.).
- Click on the cart wizard icon and generate a plot.

Your plot should look like that below.

![Landscape Evolution](image)

**Experiment with the effects of various constants.**
- **Change the diffusion constant to 10,000 km²/ma.** How does this affect landscape evolution?
- **Change \( \Delta t \) to 0.0005ma.** How does that affect the variations in profile shape you see displayed on the profile?
- **Experiment with other combinations,** but remember to keep the \( k \times \Delta t / \Delta x^2 \) term <0.5. Keep your eye on cell B4.

For today – just hand in any one of your plots. Make sure you include your name, and indicate the values of \( k \) and \( \Delta t \) used to generate the plot.
Geology 351 – Mathematics for Geologists

Computer Applications Challenges

If we have a take-home style final exam, you may be given one or more computer problems. The following are example problems. You can use PSIPlot, EXCEL or both to solve computer problems.

Problem 1. In an unconfined aquifer, pumping of a well draws the water table down into a cone of depression; water flows toward the well from the side. Given the following values and variables:

- $h_0$ = undisturbed depth to the water table = 10 m
- $r_0$ = distance to undisturbed water table = 300m and 2km (i.e. radius of cone of depression)
- $r_w$ = radius of the well, 0.1 m
- $k$ = permeability, $10^{-12}$m$^2$
- $\rho$ = density =1000 kg/m$^3$
- $Q$ = pumping rate = 25 m$^3$/day
- $\mu$ = viscosity, $10^{-3}$ kg/m-sec
- $g$ = acceleration of gravity, 9.8 m/s$^2$

Solve Equation (1) below for the depth to the water table at distance $r$ from the center of the well. Start at $r = r_w$ (i.e. the well radius of 0.1) and then compute for $r = 0.5$m, 1.0m, etc. at 0.5 meter intervals out to an $r$ of 25 meters.

The depth to the water table is approximated by the following equation -

$$h(r) = \sqrt{h_0^2 + \left(\frac{\mu Q}{k \rho g \pi}\right) \ln\left(\frac{r_0}{r}\right)}$$

(1)

Do the Following: On a separate sheet of paper show your computations of the two constant value terms in equation (1). These are -

A. $h_0^2$ and

B. Compute $\left(\frac{\mu Q}{k \rho g \pi}\right)$ and show all units in your calculation and show how these units cancel out.

Make sure you use consistent units throughout!

C. In your presentation clearly report the value of $\left(\frac{\mu Q}{k \rho g \pi}\right)$.

D. And the units of $\left(\frac{\mu Q}{k \rho g \pi}\right)$.

E. Calculate $h(r = 0.1$ meters) by hand for $r_0$ of 300 meters (show your work).

Present the above results in sections labeled A, B, C, D and E.

F. In PSIPlot or EXCEL calculate $h(r)$ for two values of $r_0$ (300m and 2km) noted above.
G. Plot both curves on one graph. Label each curve using the corresponding radius of the cone of depression (i.e. 300m or 2km).

- As always, in your own words, clearly state the problem being addressed (1 point). Include this statement at the top of your graph.
- Summarize the results of your analysis (2 points). Include this summary at the bottom of your plot.

Resize your plot so that it does not take up more than 1/2 a page. That way you will have room for the problem statement and summary along with any other additional information you wish to present. Parts A through E should be presented on a separate sheet of paper.

**Problem 2:** New ocean lithosphere is created at spreading centers. The initial temperature is around 1,350°C. Several kilometers (perpendicular to the ridge axis) of new lithosphere are created every million years. As the sea floor spreads the new lithosphere cools, contracts, and increases in density. Earth, as described by the principle of isostacy, maintains nearly equal pressure at a constant depth (the depth of compensation) over large areas. The result is that ocean depth increases as the square root of the lithosphere’s age as approximated in the following equation (2):

\[
d_{\text{ocean}} = \frac{\rho_a}{\rho_a - \rho_w} \left[ 2\alpha(T_w - T_a) \sqrt{\frac{k t}{\pi}} \right] \tag{2}
\]

In this expression,
- \(d\) = the ocean depth
- \(t\) = time in years (0 to 100,000,000 years in 10 million year increments)
- \(\rho_a\) = density of the asthenosphere = 3,300 kg/m\(^3\)
- \(\rho_w\) = density of sea water = 1030 kg/m\(^3\)
- \(\alpha\) = coefficient of thermal expansion = 3.2 \times 10^{-5}/\text{°C}
- \(k\) = thermal diffusivity of the lithosphere = 8 \times 10^{-7} \text{ m}^2/\text{s}
- \(T_w\) = temperature of the sea water \(\sim 0^\circ\text{C}\)
- \(T_a\) = Temperature of rock at the ridge crest = 1,350 °C
Do the Following:

A. Compute Ocean depth as a function of age for a 100,000,000 year time period. Make your calculations at 1 million-year time intervals. Remember $t$ in Equation 2) has units of years.

B. Plot ocean depth versus age at 1,000,000 (one million year) time intervals for the 100,000,000 year time period. Hint - generate another column of times from 0 to 100. Graph this column versus calculated depth and be sure to label the time axis as time (My).

C. Compute the change in depth between successive 1 million year intervals. Hint: copy the calculated depths into another column. Delete the zero depth from the top cell. Subtract the original column from this new column to get the differences. The differences and the depths will both be negative numbers.

D. On a separate graph, plot the change of depth versus time.

E. Experiment with titles, labels, etc., to make you plots look good.

H. Don't forget the usual - statement of the problem(s) and statement of results describing how depth and change-of-depth vary with age.
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Questions to Ponder

The following are examples of the types of questions that could be included in a written exam. On any exam, neatness and organization usually translate into a high grade. In general write answers to each question in the space provided and always show your work clearly on the work-paper provided with this exam.

1. In the diagram below, ocean floor subsidence is plotted as a function of age in millions of years. Subsidence is measured relative to the ridge crest which is considered to be at 0 depth. Determine the distance to a point on the ocean floor, which has subsided 2000 meters relative to the ridge crest. The spreading rate in this area is 3 cm per year. Assume constant spreading rate. Express your answer in kilometers. Show your calculation on the paper provided with the test.

The distance from the ridge crest is __________________.

2. Seismic moment $M$ can be expressed as $M = \mu r^3$ where $\mu$ is the shear rigidity and $r$ is the characteristic linear dimension of the surface along which fault rupture occurred. The moment-magnitude relationship is stated as $\log M = cm + d$ (where $m$ is earthquake magnitude). Evaluate $\log M$. Show the steps on the paper provided with this test.

Answer: ____________________________
3. Evaluate the $\log_2 5$. Show work on attached paper.

Answer: __________________________

4. Porosity as a function of depth is expressed as $\phi(z) = \phi_0 e^{-z/\lambda}$, where $\phi(z)$ is the porosity at depth $z$, $\lambda$ is a constant term, $\phi_0$ is the porosity at $z = 0$, $e$ is the natural base and $z$ is the depth and has units of kilometers. Given that $\phi_0$ is 0.6 and that $\phi(100\text{meters})=0.57$, a) determine $\lambda$ and b) determine the porosity at a depth of 1 kilometer. Show your work on the attached paper.

a) $\lambda =$ _______________ (include the units of $\lambda$)

b) $\phi(1\text{km}) =$ __________

5. The figure on the next page shows a highly simplified mountain range having 1 km of relief above the surrounding area. A crustal root extends 8km into the mantle lithosphere. Assume the crust has a density of 2.67 g/cm$^3$ and that the mantle lithosphere has a density of 3.3 g/cm$^3$. Determine whether the kilometer topographic relief is compensated. If it is not how deep should the crustal root be to compensate the mountain?

a) Write out the equilibrium equation in general or symbolic form.

______________________________ = _____________________________

b) Substitute values noted in the diagram below for crustal and mantle density and thickness of different zones (e.g. crust, mantle, mountain and root).

_____________________________ = _____________________________

c) Is the topographic relief compensated? ______________


d) If not, what thickness of crustal root would be required to compensate the mountain?
Answer: __________________.

e) How is the current configuration likely to evolve through time?

___________________________________________________________
___________________________________________________________
___________________________________________________________
___________________________________________________________
___________________________________________________________
Appendix I: Using PSI-Plot to solve problems 2.11 and 2.12

In this appendix, we introduce another technical plotting and scientific computing program called PSI-Plot. After gaining some experience with PsiPlot you may find it useful to use PsiPlot and Excel together in problem solving activities. The following instructions are meant to take you step-by-step through the generation and plotting of a data set. In future exercises, you will learn how to fit straight lines and polynomials of higher order to specific data sets. Today, we will take more of a conceptual look at linear relationships. We will also reformat plots into a log-log scale.

GETTING INTO PSI-Plot

PsiPlot runs in the PC windows environment. Double click on the PSI-Plot icon. When the PSI-Plot window opens up, click on FILE then on Import Data. Navigate over to the common drive (the H:/Drive) and copy the folder FittingLabData folder over to your G:/Drive. We’ll spend some time in class reviewing all this (take notes!). For today’s lab, select the data set DepthAge.dat and click the open button. A spreadsheet or window containing the depth and age data will appear on the screen.

The data in the spreadsheet were taken from Chapter 2 of Waltham’s text (page 37, problem 2.11). The problem states -

2.11 The following data were taken from the Troll 3.1 well in the Norwegian North Sea.

<table>
<thead>
<tr>
<th>Depth</th>
<th>Age</th>
</tr>
</thead>
<tbody>
<tr>
<td>19.75</td>
<td>1490</td>
</tr>
<tr>
<td>407</td>
<td>10510</td>
</tr>
<tr>
<td>545</td>
<td>11160</td>
</tr>
<tr>
<td>825</td>
<td>11730</td>
</tr>
<tr>
<td>1158</td>
<td>12410</td>
</tr>
<tr>
<td>1454</td>
<td>12585</td>
</tr>
<tr>
<td>2060</td>
<td>13445</td>
</tr>
<tr>
<td>2263</td>
<td>14685</td>
</tr>
</tbody>
</table>


For future reference, if you want to Rename a column, click on the column title, and type in the preferred column name. Hit return to enter the new name.

Follow along and **MAKE NOTES** as we go through these examples
GRAPHING AND PLOTTING!

Let’s plot up this series of numbers.

*Click on PLOT* (a menu drops down)

*Click on 2D Curve* > (menu opens to right)

*Click on XY Lines* (2D XY-Lines window opens up)

Note that in this case, X>> defaults to Depth, and Y>> to Age.

*Click on ADD CURVE>>, then OK*

Pat yourself on the back you are on your way to mastery.
**ODDS and ENDS**

Move the mouse arrow down into the plot area and *Click* it in an empty area of the plot. Note that the plot will be highlighted (e.g. square dots appear on the plot margins). You can *resize* the plot by clicking on the highlighted edge or corner points. You can move the plot around by *clicking* within the bounds of the plot and *dragging* it to a desired location. *Try it.*

To close your plot, move the mouse over to the upper left corner and *click on* the X sign. Then  
*click on* **CLOSE**, then  
*click on** **NO** (you don’t want to save it). This will return you to your spreadsheet.

**NOTES!**

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**NOTES!**
Learned one - now do one. Here are some things to try on your own.

**PLOT**

2D CURVE

XYLINES

- but, this time click on STYLE>> and select another symbol.
- Click on COLOR>> and select another color.
- Double click on the little window next to Symbol Size:, and enter 0 (only the line will appear).
- Don’t forget to click on ADD CURVE, then OK.

Note that the plots default to a landscape layout. Let’s change that to portrait.
- Click on FILE (Menu drops down - note variety of selections. Experiment later.)
- Click on Printer Setup.
- Click on Portrait, then
- Click on OK

Resize the graph to fit in the upper third of the sheet.

Double click on the graph title Sheet Untitled.
- Sheet untitled will be highlighted in blue. You can type in any title you’d like. Do so now, and also note the other options in this window, including Font, Size, Italic, etc.
- Click on OK

You can change axis labels the same way.
- Note the tool box off to the right.
- Click on the abc button
- Bring the cross hairs over to a suitable place on your graph.
- Click and drag: Hold down the left mouse button and drag open a rectangular box to place a label.
- When you release the left mouse button, a text format window will open up. You can enter a relevant label as you did for the title and axes above. Note there are lots of formatting options available to you in this window.
- Click on OK when done.

Click on your label, and move it around.

Click on an open space within the graph. Note that the graph is highlighted. Now
- Click on the label you entered above. Note that the graph remains highlighted.
- In the newer releases this is not a problem, but if you should have problems re-editing your label you may have to push the graph or other active object “back.” To do this -
- Go to VIEW (a window drops down).
- Click on Push Back. (This will push the active window back and give you access to other foreground plot elements.)
- Now click on your label. There you go!
**Back to Problem 2.11:** From the graph and data listing (i) estimate the sedimentation rate for the last 10000 years, (ii) the sedimentation rate for the preceding 5000 years, and (iii) the time since sedimentation ceased.

Compute $\Delta Depth \over \Delta Age$ from data points with depths extending from 19.75 cm to 407 cm and ages of 1490 years to 10510 years. This yields

$$\frac{387.25}{9020} = 0.043 \text{ cm/year (or } 23.29 \text{ years/cm)}.$$
GENERATING A LOG-LOG PLOT-

Remember from our in-class discussions of possible age depth relationships we suggested that the result of compaction and porosity reduction could be variable through time and have the effect of increasing the amount of time represented by a unit thickness (for example, a meter or ten meters) of strata at greater and greater depths of burial. We might suspect that age might increase non-linearly with increasing depth, and that since porosity decreases exponentially with depth in many sedimentary deposits, perhaps age increases exponentially with depth such that \[ A = A_0 e^{\alpha d} \].

As we do this though, think what you would get if you took the natural logarithm of both sides of this equation. What kind of equation results from this operation?

Let's check it out.

Double Click on the x axis (Depth axis in this case). An Axis Format window will open up. Go over to the axis mode, click on the down arrow, and change linear norm to ln (natural). Click OK. Note that the Age axis will be rescaled into logarithmic (base e) intervals.

Does your plot look more like a straight line?

This has just been a basic run through on some of the options available through PLOT. We will return for more later in this DEMO. But the best way to learn will be to experiment.

MORE NOTES?
Saving your data.

Once you’re finished you can save your data. I suggest that you save your data in the **g:drive**.

**Take Notes on how to do this!**

*Click on File - Save As*

*And select the g:drive from the saving plot window.*

Give your file a name you will remember like problem2-11. I suggest using a - or an underscore _ instead of a . since the computer will consider words separated by periods as extensions.

When you save on the g:drive you can go to another computer and access your data. If you save on the c:drive (the default save drive) you will only be able to access your data from that machine.

Plot files have a PGW extension (G for Graphics), while data files have a PDW extension (D for Data).

Save your plot, close the plot window (click on the x in the upper corner of your plot window -NOT THE PSIPLLOT WINDOW!!

You should now be back in your spreadsheet. Also save it also and close.

**MORE NOTES**
Problem 2.12
As crystals settle out of magmas, the element concentrations (C in formula below) in the remaining liquid change according to the equation

\[ C = C_0 F^{(d-1)} \]

where \( C_0 \) is the initial concentration of the element in the liquid before crystallization began, \( F \) is the fraction of liquid remaining and \( D \) is a constant (known as the distribution coefficient). Calculate the concentration of an element after 50\% crystallization (i.e. \( F = 0.5 \)) if its initial concentration was 200ppm and \( D=6.5 \).

Let's take a different approach to the solution of Problem 2.12. Rather than solving \( C \) for just one value of \( F \) let's solve \( C \) for a range of \( F \)s extending from 0 to 1 at intervals of 0.05.

This will give us a total of 21 computations of \( C \). Sounds like a lot of work, but with the help of PsiPlot, we can probably do all that in the time it would take you to do one computation by hand.

First -
Open up a blank spreadsheet. Click on the short-cut button just below the File option (see right)

This will open up a brand new spreadsheet.

Now we want to generate a column of numbers corresponding to \( F \) (the fraction of liquid remaining) that range from 0 to 1 in increments of 0.05. To do this, click on Data, Fill Selection and then Algebraic (see illustration at right).

Another window will pop up (see right). We are placing values of \( F \) in column 1. The values will occupy cells 1 through 21. The first cell has a value \( F = 0 \) and each consecutive cell will have a value \( F \) incremented by 0.05.
Rename that column F so that your spreadsheet looks like that right.

Now go to **Math Transform** (right) and enter the equation

\[ C = 200 \times F^{5.5} \]

* represents the multiplication operator

^ represents the exponentiation or power operator

The equation says

\[ C = C_0 F^{(D-1)}. \]

What would you get if you took the natural log of \( C \) (i.e. \( \ln(C) \))? What would you get if you took the base 10 log of \( C \) (i.e. \( \log(C) \))?
Hit the enter key and the column next to $F$ will be filled with values and labeled $C$ as shown at right.

How does concentration ($C$) vary with liquid fraction ($F$)?
Remember how to generate a plot?
Appendix II: Quadratics and Settling Velocities using PsiPlot

In Chapter 3 of *Mathematics: A simple tool for geologists*, Waltham takes us through a brief review of quadratic equations and their roots. In the computer lab today, we explore the graphical significance of roots and use PsiPlot to help us solve problems at the end of Chapter 3.

GET INTO PSI-Plot

**PART I:** Graph the following functions:

D) \( y = 3x^2 - x - 5 \)
E) \( y = x^2 + x + 3 \)
F) \( y = -x^2 + 2x - 1 \)

and determine their roots.

Open up a blank spreadsheet. Click on the short-cut button just below the File option (see right).

This opens up a new spreadsheet.

Now we want to generate a column of numbers corresponding to the x's in the above equations. The roots of these equations are evident over the interval -2 < x < 2. So we need to **FILL** column 1 with x values that run from -2 to 2. One also needs to decide on a computation interval. In this case let's use 0.1 … So click on Data, Fill Selection and then **Algebraic** (see illustration at right).

The **Fill Selection window** pops up (see right). Values of x running from -2 to 2 will occupy 41 cells (rows) - 1 through 41. The first cell has a value -2 and each consecutive cell will have a value x incremented by 0.1.
Rename that column to X so that your spreadsheet looks like that shown at right.

Now go to Math Transform (right) and enter the equation \( y = 3x^2-x-5 \)
Recall we must use the proper operator notation. So enter y=3\(x^2\)-x-5, as shown at right.

Remember -
* represents the multiplication operator
\(^\) represents the exponentiation or power operator
Hit the enter key and the column next to $X$ will be filled with values and labeled $Y$ as shown at right.

What does this quadratic equation look like?
Remember how to Generate a plot?

Go to Plot 2D curve XY Lines

The window at right will pop up.
$X$ and $Y$ are the only two variables in the list and they are the default $X$ and $Y$ plot variables. Just click on Add Curve and your plot will appear.
Practice your plot formatting skills.

Where would you guess the roots of the equation $3x^2-x-5$ are located?

Generate plots of quadratics B and C.

D) $y=x^2+x+3$
E) $y=-x^2+2x-1$

PART II:

Problem 3.11 Stokes’ law states that the viscosity at which a spherical particle suspended in a fluid settles is given by

$$v = \frac{2(\rho_p - \rho_f)gr^2}{9\eta}$$

where $v$ is the velocity of descent, $\rho_p$ and $\rho_f$ are the densities of particle and fluid respectively, $g$ is the acceleration due to gravity, $r$ is the particle radius and $\eta$ is a property of the fluid known as viscosity. Assuming that grains of different sizes have identical densities, show that the ratio of the settling velocities for two different grain sizes is

$$\frac{v_1}{v_2} = \left(\frac{r_1}{r_2}\right)^2$$

where $v_1$ and $v_2$ are the velocities for grains of radius $r_1$ and $r_2$ respectively. If a grain of radius 0.1 mm, suspended in a lake takes 10 days to settle to the lake bottom, how long would it take a grain of radius 1 mm.
Viscosity is a measure of the resistance of a liquid to flow. The viscosity of water at room temperature is about 0.01 poises. 1 poise is 1 gram/cm-second. The units of viscosity are also often given in pressure-seconds such as Pascal-Seconds (a pascal is one Newton/meter²). A thick oil might have a viscosity of about 1.0 poise.

Using a viscosity of 0.01 poise (gm/cm-s) for the settling of different size sand grains in a lake. Let the particle sizes range from 0.001 cm to 0.1 cm and increase the particle radius by increments of 0.001 cm over the range. Use \( g = 980 \text{cm/sec}^2 \), \( \rho_{\text{sand}} = 2.67 \text{gm/cm}^3 \), \( \rho_{\text{water}} = 1 \text{ gm/cm}^3 \).

Use the Data Fill Selection - Algebraic option to generate 100 values of \( r \) in column 1 that range from 0.001 to 0.1 and relabel the column to \( r \). Then use the Math Transform option to generate the velocities. Your equation should look like that at right.

Plot your data. You should get a plot that looks like the one below.

- **Answer this question**: Determine the velocity with which a grain with radius 0.1mm settles in the lake. If it takes the grain 10 days for the grain to settle to the bottom, how deep is the lake?
Given that the time it takes a particle to settle to the bottom is equal to depth/velocity, construct a plot of settling time versus particle size using a lake with depth equal to 100 meters and the same range of particle sizes used in the preceding example. Note that you need to consider the units in the numerator and denominator. If the units of velocity are in cm/s what should the units of depth be?

Part 1 is just a learning activity. You do not need to hand this in. If you do I'll be happy to look it over.

1) Plots of quadratic formulas A, B and C. Label locations of the roots on each graph. In a sentence or two, comment on the solutions shown in each of your graphs. Include your written comment on the graph.

Work to be handed in for this lab assignment:

2) Hand in a plot of settling velocity versus particle radius. Leave space on the plot so that you can include computations of the lake depth and a short discussion of what you did and why.

3) Hand in a plot of settling time versus particle radius. Comment on how the plot of settling time compares to the plot of settling velocity.

Be prepared to ask questions this Thursday. This homework will be due on Tuesday, March 13th at the beginning of class.

Problem 3.10 is also due next Tuesday. Since this is worked in the back of the text, be sure to explain each step. Don’t just copy the answer from the back!

Remember – well organized presentations and discussion are critical to making sense out of ones results.

This homework will be due ________________ at the beginning of class so we can discuss results and move onto new material.
Appendix III: Analyzing Pump-Test data in PsiPlot

The following illustrated discussions take you through each step of the calculations for analysis of the drawdown phase behavior and computation of the maximum sustained yield using PSIPlot. Refer back to the introductory materials in the discussion of the Lab 5 (Estimating Maximum Sustained Yield) for background information on the pump test calculations and relationships.

To Start: Open up a PSIPlot spreadsheet and enter the above times and water depths. Label your columns time and depth. Next transform time to \( t \) (in days) and water depths into actual drawdowns. Transform time in minutes to time \( t \) in days by dividing time by 1440 (the number of minutes in a day. The time in days is listed as \( T \) in the PsiPlot worksheet shown at the top of the next page. The actual drawdown is water depth minus the initial water depth. Use the Transform option as illustrated in the following figure.

In the MATH TRANSFORM Equation \( DD = D - 76.21 \), \( DD \) stands for the drawdown (or \( s_u \) in the above descriptions of terms). Note that \( D \) also incorporates the additional height above ground surface to the top of the casing (+2.1 feet in this case).
Use the **transform** option again and take the $\log_{10}$ of the pumping time in days (i.e. $\log_{10}(T)$). PsiPlot's $\log_{10}$ operator is just $\log_{10}$. So your transform operation will look something like that at right. Emphasize that **$T$ is the time in days**.

Your spreadsheet should now have the following columns of data (see below). Note that the first cell in the column **logT** is blank since the log of 0 is undefined.
Next compute the corrected drawdowns ($s_a$), where $s_a = s_u - s_u^2/2m$. In this problem $m$, the static saturated aquifer thickness ($m$) was found to be 229 feet. Depending on your own notation, the *math transform* equation will look like that below.

The pumping operation lasted for 60 minutes. Hence water table recovery begins at time = 60 minutes. $t'$ the time since pumping stopped will equal time-60 for all observation times following 60 minutes. To undertake the estimate of transmissivity from the recovery data, copy all times of 60 minutes and greater from column 1 to column 6 and label that column $T_{Rec}$.

Then use Math Transform and subtract 60 from $T_{Rec}$ to obtain $T'$ (i.e. $T' = T_{Rec} - 60$). Your spreadsheet should look like that below.

Finally, compute the log$_{10}$($T_Rec/T'$) (i.e. LOGTRAT=LOG10($T_{Rec}/T'$). This variable (I called it LOGTRAT (log of the ratio of times) is required in the analysis of recovery stage data. Don't forget that the log$_{10}$ operation is performed using the PSIPlot operator LOG10. Your spreadsheet should now look like that below.

---

### Table 1

<table>
<thead>
<tr>
<th>Column</th>
<th>D</th>
<th>T</th>
<th>DD</th>
<th>LogT</th>
<th>S</th>
<th>T_{Rec}</th>
<th>T'</th>
<th>Transform</th>
<th>C10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0000</td>
<td>76.2100</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>60.0000</td>
<td>0.0000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
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<td>0.0005</td>
<td>12.5300</td>
<td>-2.4554</td>
<td>12.5550</td>
<td>63.0000</td>
<td>3.0000</td>
<td>1.3222</td>
</tr>
<tr>
<td>3</td>
<td>0.0000</td>
<td>93.6900</td>
<td>0.0005</td>
<td>17.7100</td>
<td>-2.2553</td>
<td>17.0250</td>
<td>69.0000</td>
<td>3.0000</td>
<td>1.3222</td>
</tr>
<tr>
<td>4</td>
<td>10.0000</td>
<td>96.7900</td>
<td>0.0005</td>
<td>29.5700</td>
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You are now ready to plot the drawdown data and undertake the slope analysis.

**First, let’s work with the pumping stage data.**
Generate a plot of the variables $\text{LOGT}$ vs. $S$. Your plot should look similar to that shown below.

The pumping and recovery phases of the water table response are easily identified in the above plot. The slope ($\Delta S/\Delta \log_{10}t$) should be calculated in the highlighted linear region (see highlighted points in figure above).

**Next**, you will have to isolate the data in this region of the plot for analysis. The region of data you want to fit a straight line to correspond to the last 7 observations made during the pumping stage at times corresponding to 41 minutes through 60 minutes. Highlight that range of values (see right) and copy to unfilled columns in your spreadsheet.
Label these new columns LT and SD (see figure below).

The previous computer lab handout illustrated the regression line estimation procedures for several classes of curves. In the present example, we assume the response is linear and compute the slope of a line fit to this region of the drawdown. The quickest way to do this is to click on the linear regression shortcut icon circled below.

In the drop down menu specify the x and y variables as LT and SD respectively (see below). LT and SD are just the column labels I used to denote the copied range of LOGT and S data toward the end of the drawdown phase (41 through 60 minutes).
Then click OK.

The **Linear Regression Report** window will appear (see right). Note that \(a\) is the slope of the fitted line. You should write down the value of \(a\), which in this case is **37.69 feet/day**.

Now we’re ready to compute the transmissivity \(T\) using equation 3) \(T = 264Q/(\text{slope})\) which has units of gal/d/ft or from equation 4) \(T = 35.3Q/(\text{slope})\), which has units of ft\(^3\)/d/ft. We were given \(Q\) above as 13.8 gallons/minute. Remember that equations 3) and 4) have been set up so that we can use \(Q\) in gallons/minute and slope in feet, so no additional units conversions are necessary. Plugging values for \(Q\) and the slope (37.69 feet) into equation 3), we get \(T = 96.66\text{(gal/d)/ft}\). Plugging into equation 4), we get \(T = 12.92\text{(ft}^3\text{/d)/ft}\).

The hydraulic conductivity \(K\) (=T/m) (Equation 5) is just 12.92/229 or 0.057 ft/d or in units of gpd/ft\(^2\) **K=96.66/229 or just 0.42 gpd/ft\(^2\)**.

The maximum sustained yield is calculated using Equation 7) -

\[ Q = K(H^2 - h^2)/[1055\log_{10}(R/r)] \]

Equation 7) provides maximum sustained yield in units of gpm, and assumes that \(K\) is reported in units of gpd/ft\(^2\). hence we use \(K =0.42\text{gpd/ft}^2\) as derived above. Recall that \(H = m = 229\text{feet}\). \(h\) is the assumed height of the water column above the base of the well during maximum sustained pumping, which was assumed by Dr. Rauch to be 20 ft; \(R\) is the radius of the steady state pumping cone of depression in the water table. Values of \(R\) are actually unknown, but based on experience are believed to fall between 100 and 1000 feet. Dr. Rauch calculate \(Q\) using three values for \(R\); \(R=100\), \(R=300\) and \(R=1000\) feet. \(r\) is the well radius below the casing, which at this site was 3.5 inches or 0.292 feet.

\[ Q = 0.422(229^2 - 20^2)/[1055\log_{10}(300/0.292)] = 6.91\text{gpm} \] using an \(R\) of 300 ft.

For illustration, we will restrict ourselves to one calculation; however, separate calculations for \(R\) of 100 feet and 1000 feet provide perspective on the possible range of sustained flow that might result. Dr. Rauch reports sustained flows that range between 5.9 and 8.2 gpm for \(K\) estimated from the pumping phase drawdown response.
Secondly, let's estimate $T$ from the recovery stage data. Estimates made from the pumping and recovery phases of drawdown observations provide slightly different values of $T$ and $K$. The true transmissivity and hydraulic conductivity probably lies somewhere between these two estimates. Computations of $T$ and $K$ are usually made from both the drawdown and recovery phases to provide perspective on the range of possible sustained yields that may occur over the actual lifetime of a particular well.

In the remainder of this lab, we estimate $T$ using Equation 6)

$$T = 2.303Q\left(\frac{\log_{10} t_2}{t_2} - \frac{\log_{10} t_1}{t_1}\right)/4\pi(s_2 - s_1).$$

Note that in this equation the slope is derived from $s$ versus $\log(t/t')$ data. Hence your first task will be to construct a plot of $s$ versus $\log(t/t')$ obtained from water depths recorded following the cessation of pumping.

At this point, you can delete columns LT and SD along with the regression line columns (Ind12 and Dep13) from columns 12 and 13 of the PsiPlot spreadsheet. We don’t need these data any more.

Recall that we have already computed the log-transformed time ratios (LOGTRAT in spreadsheet view below). However, we will have to copy the corresponding range of corrected drawdown values ($S$ in spreadsheet below) into a separate column for plotting and line fitting purposes. Do that now and give that column of data the label SRec.

NEXT - plot SRec versus LOGTRAT. Your plot should like that shown below. Adjust axis labels for presentation and discussion.
Now go back to your spreadsheet and select the last 4 rows of LOGTRAT and SR values (LOGTRATs in the range ~ 0.39 to 0.3). Note that these cells are located at the bottom of the LOGTRAT column. Copy these 4 rows into two blank columns off to the right. Do the same for the corresponding values of SRec. Label these two new columns RX and SY (see below).

![Graph with annotations](image)

You do not have to plot the fitted line on the graph. It’s great if you do, and will help verify that you have done this correctly. The red line at right was added using the line drawing tool.

Note that the smallest values of log(t/t') correspond to the longest drawdown times (TR).

Finally, fit a straight line to RX and SY. In the line fitting summary window write down the slope you obtain. This lab guide carries you through the analysis right up to the final calculation steps.
Use the following equations to solve for T (restated from page 58).

\[ T = \frac{264Q}{(slope)} \quad [(\text{gal/d})/\text{ft}] \quad 3 \]  
\[ T = \frac{35.3Q}{(slope)} \quad [(\text{ft}^3/\text{d})/\text{ft}] \quad 4 \]

to get the transmissivity in the desired units.

- Compute K in gpd/ft² (use equation 5)
- Determine the maximum sustained yield Q (gpm) assuming an R of 300 feet. Use equation 7

**Assignment Check List**

10. Hand in a plot of the Pump Test data (see page 64). Label the pumping and recovery phase portions of the data. Place a caption on the figure to indicate what the figure represents. Place your name in the plot title.
11. Summarize results (T, K, and Q) obtained from the pumping stage.
12. Prepare and hand in a plot of the recovery phase data (LOGTRAT vs. sₐ, see top of page 68). Note the slope you obtained. Add appropriate labels and caption to the figure for clarification. Place your name in the plot title.
13. Determine T in units of \([(\text{gal/d})/\text{ft}] \) (use equation 3 above). Show your calculations and state what it is you are presenting.
14. Compute Hydraulic conductivity K in units of gpd/ft². Show your calculations and state that this is what you are doing.
15. Solve for Q (gpm) using equation 7 for R= 300 feet. Show calculations and state results.
16. Clearly show your calculations for transmissivity, conductivity and sustained yield.
17. In summary, list your results and clearly indicate the values obtained for transmissivity, conductivity, and sustained yield. Compare results obtained from the pumping and recovery stages of the test.

Always present your results in an organized fashion; clearly label and reference figures in your work statements.