We'll take a quick look at an additional example of math applications in geology – our last one for the semester! This is another geomorphology example, and deals with the evolution of landscapes through time.

**Landscape Evolution**

Along with the preceding erosion problem the following problem is also taken from Steve Sheriff's site at http://www.cs.umt.edu/GEOLOGY/classes/Comp_Geol/compgeol.htm. Also note that Charlie Angevine (University of Wyoming) suggested the ideas presented by Sheriff. The background for the computational approach to landscape evolution illustrated here comes from Slingerland, Harbaugh, and Furlong (*Simulating Clastic Sedimentary Basins* (1994, Prentice Hall, 220p)).

The erosion problem gave us an appreciation for the time frame over which landscape features will erode. In the current problem we consider how landform shape will evolve through time. The basic equation governing this process is the diffusion equation –

\[
\frac{dh}{dt} = k \frac{d^2h}{dx^2}
\]

where \(h\) is the elevation, \(dx\) is differential distance along the profile, \(k\) is a diffusion constant, and \(dt\) is a small span of time during which we examine the change in landform shape.

Note that there is a second derivative of elevation in the equation with respect to the spatial position on the line. First we compute the change in elevation at a given point along the profile. This is just the gradient or slope of the surface. We next compute the change in the slope with distance. That is the second derivative. So if the slope is changing rapidly –either rising toward the top of a hill or dropping into a valley, the second derivative will be large. In these areas, one will either have the greatest erosion rates with time or the least, respectively. In the later case, we have assumed that there is no transport, and that internal valleys within the profile will fill up.

The equation represents a condition in which changes in the elevation of a point along a profile are proportional to changes in slope bordering that point and to the time span over which erosion occurs. To compute how a profile will evolve, we must first compute \(\Delta h\) for a certain time period \(\Delta t\) and then subtract or add \(\Delta h\) to the original profile elevation. So that we can compute these changes on the computer, we look at sizable, but still small changes in time space and elevation. Thus, from equation 1, our computer computation starts with.
\[
\frac{\Delta h}{\Delta t} = k \frac{\Delta (\Delta h)}{\Delta t \Delta x}
\]
which becomes \[
\Delta h = k \frac{\Delta}{\Delta x} \left[ \frac{\Delta h}{\Delta x} \right] \Delta t
\]

To begin our computation we compute the inside term, \[k \left[ \frac{\Delta h}{\Delta x} \right],\] first. Let’s call this term \(G\), which is basically the gradient scaled by the diffusion constant. Then we compute the spatial change in this quantity over a certain time period: i.e.,

\[
\Delta h = \frac{\Delta G}{\Delta x} \Delta t
\]  \hspace{1cm} (2)

Before we go undertake this computation on Excel, let’s look at the assumptions made by this equation. As always, the equations you use will have certain assumptions that go along with them. As Sheriff notes, in this case, the following assumptions are being made:

- We approximate a smooth topographic profile with a series of adjacent spreadsheet cells, each holding the average elevation of the surrounding area.
- We assume that the material being eroded, transported and deposited in and out of each cell is homogeneous,
- We assume that the rivers at the edge of our landscape transport all the debris of erosion supplied to them and that they do not incise over time.
- We assume a two-dimensional landscape with no transport in and out of the plane of the cross section.
- We make no distinction between the different types of erosion, but consider only their long-term net effects.
- We assume the long-term average flux of materials on a slope is proportional to the slope.

Let’s set up the computational procedures.

As always, we have certain constants we can place in a prominent position within the worksheet and make absolute references to in our computations. The constants in the current application will be the diffusion constant \(k = 20,000\text{km}^2/\text{ma}\), \(\Delta x = 10\text{km}\), and \(\Delta t = 0.001\text{ma}\). Also insert the computation \(k^*\Delta t/\Delta x^2\) into one cell and label it. After the computational procedures are all set up, you can change the values of \(\Delta x\), \(\Delta t\), and \(k\) to determine their relative effect on profile erosion, but you will want to be sure to keep the \(k^*\Delta t/\Delta x^2\) term <0.5. When this term exceeds 0.5 you will see some interesting effects.

Next, we must specify our topographic profile. For the purposes of illustration, I will use the same profile that Sheriff does in his example, but you can change this around on your own, if you are interested to see how different profile shapes erode.

The starting profile has the following elevations in kilometers spaced at 1km intervals:
0.0, 0.5, 1, 2, 3, 4, 4, 3, 2, 2, 1, 1.5, 1.5, 2, 1.5, 1, 0.5, 0.0
At this point, your spreadsheet may look like that shown below.

Let's look at how we can code in the computation of \( k \left[ \frac{\Delta h}{\Delta x} \right]. \) In the spreadsheet you have sample distances in row 5, cells B5 through T5. The starting elevations are listed below in cells B6 through T6. To compute the term \( k \left[ \frac{\Delta h}{\Delta x} \right] \) for the elevation in cell G6 we do this by incorporating the slope to the left and right of the point. So we need actually to compute two terms: \( \left[ \frac{\Delta h}{\Delta x} \right]_{\text{LEFT}} \) and \( \left[ \frac{\Delta h}{\Delta x} \right]_{\text{RIGHT}}. \) Since erosion generally reduces the elevation these terms are negative, and we need to tell Excel to compute

\[
\Delta G = \left( -\left( \frac{k}{\Delta x} \right) \Delta h_{\text{LEFT}} - \left( \frac{k}{\Delta x} \right) \Delta h_{\text{RIGHT}} \right),
\]

hence the Excel formula looks like:

\[((-B3/B1)*(G6-F6))+(-(-B3/B1)*(G6-H6)).\]

We complete the computation by carrying out the calculation shown in equation 2:
\[ \Delta h = \frac{\Delta G}{\Delta x} \Delta t. \]

The completed Excel formula looks like this:

\[ = G6 + (((-B3/B1)*(G6-F6))+(-B3/B1)*(G6-H6)) / B1 \times B2. \]

(Don’t confuse G in the Excel formula with \( G \), the gradient. \( \Delta x \) is in cell \$B\$1, and \( \Delta t \) is in cell \$B\$2. Recall that what we have computed is the erosion of material out of the section, hence we add the result to the elevation in the previous step.

**Today’s Class Exercise**

- Cut the highlighted green cells in row 7 and paste in the next 70 to 80 lines below.
- Select the distances along the profile in row 6, the initial topographic relief in row 7 (remember to hold down on the Ctrl key when you are doing this), then select every 10\(^{th}\) row down to the end of your data set (e.g. rows 16, 26, 36, …, etc.).
- Click on the cart wizard icon and generate a plot.

Your plot should look like that below.

![Landscape Evolution Graph](image)

**Experiment with the effects of various constants.**

- Change the diffusion constant to 10,000 km\(^2\)/ma. How does this affect landscape evolution?
- Change \( \Delta t \) to 0.0005ma. How does that affect the variations in profile shape you see displayed on the profile?
- Experiment with other combinations, but remember to keep the \( k*\Delta t/\Delta x^2 \) term <0.5. Keep your eye on cell B4.

**For today – just hand I any one of your plots. Make sure you include your name, and indicate the values of \( k \) and \( \Delta t \) used to generate the plot.**