Calculus V

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Discussion: Chapter 8 Problems

Questions?
### Derivatives

\[ \frac{dx}{d\theta} \]

**Forwards & Backwards & forwards**

| Group 1 | \[
\frac{\sin(\theta)}{e^x + x^2 - \sin(x)}
\]
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<tbody>
<tr>
<td>1. (\cos(\theta))</td>
<td>(\frac{\sin(\theta)}{e^x + x^2 - \sin(x)})</td>
</tr>
<tr>
<td>2. (e^x + 20x^2 - \cos(x))</td>
<td>(e^x)</td>
</tr>
<tr>
<td>3. (ax^2)</td>
<td>(\sin(x) - \frac{x^3}{3})</td>
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<td>4. (\cos(x) - x^3)</td>
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**Review**

**The indefinite integral**

\[ \int nt^2 dt = \frac{nt^2}{2} + C \]

**Graphically, what does this represent?**

Basically it says the starting point can vary. To know where the thing is going to be at a certain time, you have to know where it started. You have to know \(C\).
The velocity of the object doesn't depend on the starting point;
but - location as a function of time obviously does depend on the starting point.

\[ \frac{nt^2}{2} + C \]

There’s another class of integrals in which the limits of integration are specified, such as

\[ \int_0^T nt \, dt = \frac{nt^2}{2} \left. \right|_0^T \]

This is referred to as the definite integral and is evaluated as follows

\[ \int_0^T nt \, dt = \frac{nT^2}{2} - \frac{n0^2}{2} = \frac{nT^2}{2} \]
& in general ...

\[ \int_a^b nt^2 dt = \left. \frac{nt^2}{2} \right|_a^b = \frac{na^2}{2} - \frac{nb^2}{2} \]

Those constants cancels out.

\[ y = \int_1^2 xdx = \left[ \frac{x^2}{2} + k \right]_1^2 = \left[ \frac{2^2}{2} + k \right] - \left[ \frac{1^2}{2} + k \right] \]

\[ 2 - 0.5 + k - k \]

You have to have additional observations to determine \( C \) or \( k \).

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**Another example of a definite integral**

What is the area under the cosine from \( \pi/2 \) to \( 3\pi/2 \)

\[ \int_{\pi/2}^{3\pi/2} \cos(a)da \]

Before you evaluate this, draw a picture of the cosine and ask yourself what the area will be over this range

\[ [\sin(a)]_{\pi/2}^{3\pi/2} \]

\[ = \sin(3\pi/2) - \sin(\pi/2) \]
Some elementary integration rules

**Multiplication rule for integrals**

Given

\[ \int af(x) \, dx \]

where \( a \) is a constant; \[ a \int f(x) \, dx \]

*e.g.* \[ \int 3 \sin(x) \, dx = 3 \int \sin(x) \, dx \]

\( a \) cannot be a function of \( x \).

**Integral of a sum**

\[ \int [f(x) + g(x) + h(x)] \, dx \]

\[ = \int f(x) \, dx + \int g(x) \, dx + \int h(x) \, dx \]

**Power rule for integrals**

\[ \int x^n \, dx = \frac{1}{n+1} x^{n+1} + C \]
What is the volume of Mt. Fuji?

\[ V = \sum_{i=1}^{N} \Delta V_i = \sum_{i=1}^{N} \pi r_i^2 \Delta z \]

\[ V = \int_{z_{\text{min}}}^{z_{\text{max}}} \pi r_i^2 \, dz \]

\[ \int 3t^2 \sqrt{8 + 3t^3} \, dt \]
\[ \pi r_i^2 \, dz \]
is the volume of a disk having radius \( r \) and thickness \( dz \).

\[
V = \int_{z_{\text{min}}}^{z_{\text{max}}} \pi r_i^2 \, dz = \text{total volume}
\]
The sum of all disks with thickness \( dz \)

Waltham notes that for Mt. Fuji, \( r^2 \) can be approximated by the following polynomial:

\[
r^2 = \frac{400z}{3} - \frac{800 \sqrt{z}}{\sqrt{3}} + 400 km^2
\]

To find the volume we evaluate the definite integral:

\[
V = \pi \int_0^3 \frac{400z}{3} \, dz - \pi \int_0^3 \frac{800 \sqrt{z}}{\sqrt{3}} \, dz + \pi \int_0^3 400 \, dz
\]
The “definite” solution

\[ V = \int_{0}^{3} \pi r^2 \, dz \]

\[ \pi \left[ \frac{400z^2}{6} - \frac{800z^{1.5}}{1.5\sqrt{3}} + 400z \right]_{0}^{3} \]

\[ \pi (600 - 1600 + 1200) \]

\[ = 200\pi = 628\, km^3 \]

Example 9.7 - find the cross sectional area of a sedimentary deposit (see handout).
\[ \int_0^x (ax^2 + bx + c) \, dx = \frac{ax^3}{3} + \frac{bx^2}{2} + cx + C \]

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<td>1</td>
<td>1</td>
<td>x</td>
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\[ t = -2.857E-12x^4 + 1.303E-08x^3 - 2.173E-05x^2 + 1.423E-02x - 7.784E-02 \]

\[ t = \sum \]

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<td>1/2</td>
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<td>-1.56E+02</td>
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\[ t = \sum \]

\[ t = 4073.73 \]
1. Hand integral worksheets in on Thursday
2. Bring questions in about problem 9.7 this Thursday.