The total natural strain, $\varepsilon$, is the sum of an infinite number of infinitely small extensions.

In our example, this gives us the definite integral:

$$\varepsilon = \int_{L_i}^{L_f} \frac{dL}{L} = \ln \left( \frac{L_f}{L_i} \right)$$

$$= \ln(L_f) - \ln(L_i)$$

$$= \ln \left( \frac{L_f}{L_i} \right)$$

Where $S$ is the Stretch
We can simplify the problem and still obtain a useful result.

Approximate the average densities

\[ 11,000 \text{ kg/m}^3 \]
\[ 4,500 \text{ kg/m}^3 \]

\[ M = \sum_{i=1}^{n} 4\pi r_i^2 \rho_i \Delta r \]
\[ M = \int_0^R 4\pi r^2 \rho \, dr \]

The result – 6.02 x 10^{24} \text{ kg} is close to the generally accepted value of 5.97 x 10^{24} \text{ kg}.
Where $z$ is the distance in km from the base of the Earth’s crust and $Q$ is heat per unit volume

i. Determine the heat generation rate at 0, 10, 20, and 30 km from the base of the crust

\[
\frac{0 \text{ kW}}{20 \text{ km}} = 0 \frac{\text{kW}}{\text{km}^3} \quad \frac{10 \text{ kW}}{20 \text{ km}^3} = 0.5 \frac{\text{kW}}{\text{km}^3}
\]

ii. What is the heat generated in a small box-shaped volume $\Delta z$ thick and 1 km x 1 km surface? Since

\[
\Delta V = \Delta z \quad \Delta V = \Delta z \text{km}^3 \quad Q \Delta V = Q \Delta z \text{km}^3
\]

Units

Energy is often expressed in joules, where 1 joule is 1Nt-m (newton meter).

One joule/sec is a unit of power (the rate at which energy is expended/supplied).

One joule/sec is a watt.
http://www.onlineconversion.com/

There you will find that one kilowatt corresponds to 1.34 horsepower. So your hundred watt light bulb gives off the heat of a little more than 1/10th of the horsepower.

The heat generated will be

\[ Q \Delta V = Q \Delta z \ kW \]

This is a differential quantity so there is no need to integrate iii & iv. Heat generated in the vertical column

In this case the sum extends over a large range of \( \Delta z \), so

However, integration is the way to go.

\[ iii. \ q = \sum_{i=1}^{n} Q_i \Delta z \]

\[ iv. \ q = \int_{0}^{1} z \ dx = \frac{1}{20} \int_{0}^{1} z \ dx \]
v. Determining the flow rate at an elevation $z$ above the base of the crust would require evaluation of the definite integral

$$q = \int_0^z Qdz$$

$$q = \frac{z^2}{40} \bigg|_0^{10} = 22.5 kW$$

vi. To generate 100MW of power

$$\frac{100,000 kW}{22.5 kW/km^2} = 4444 km^2$$

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**Problems 9.9 and 9.10?**

Volume of the earth - an oblate spheroid

$$r^2 = r_e^2 \left( 1 - \frac{z^2}{r_p^2} \right)$$

In this equation $r$ varies from $r_e$, at the equator, to $r=0$ at the poles. $z$ represents distance along the earth’s rotation axis and varies from $-r_p$ to $r_p$.

The equatorial radius is given as 6378km and the polar radius, as 6457km.
9.9

\[ r^2 = r_e^2 \left(1 - \frac{z^2}{r_e^2}\right) \]

i. Volume of a disk \( \Delta z \) thick.

ii. \[ \sum \] Area of disk times its thickness

iii. Rewrite the discrete sum as an integral

iv. Given equatorial and polar radii of 6378km and 6357km, respectively, what is the volume of the earth?

v. How does the result compare to the volume of spheres with radii of 6378km and 6357km

9.10

Recall the relationship defining the thickness of the bottomset bed as a function of distance from its onset.

\[ t = t_0 e^{-\gamma x} \]

i. How could you calculate the cross-sectional area of the bottomset bed using a discrete sum of small rectangles \( \Delta x \) wide.

ii. Write down the sum

iii. Evaluate the indefinite integral

iv. Evaluate the definite integral
Examine the map and measure the distance from the church to the transmitter. If from an exposure, the church is seen to be located 45° west of north whilst the transmitter is due west, where is the exposure? How far is the exposure from the church and how far from the transmitter? For today, assume church is due north of the transmitter to make this approximation.

Given the outcrop width of 1.25 kilometers for this massive sand which dips northwest at 27°...

.. what is the true (bed normal) thickness (T) of the sand?

\[
\text{Thickness } T = W \sin(27) \\
T = 1.25 \times 0.45 \\
T = 0.57\text{km}
\]
IN-CLASS PROBLEMS

1. At the base of a cliff you are standing on top of geological Unit A. The cliff face is formed along a normal fault (nearly vertical). The top of Unit A is also exposed at the top of the cliff face. You walk a distance $x = 200$ feet away from the fault scarp. Looking back toward the cliff, you use your Brunton and measure and note that the top of the cliff is $23^\circ$ above the horizon. What is the offset along this fault?

![Diagram showing cliff face, top of Unit A, and a measurement setup]

2. A group trekking through the Himalayas quickly gets lost having forgotten their top maps. They did bring their radio transmitter though.

How can they help the rescue team determine their location? Assume they have a digital altimeter/barometer and all-purpose Brunton compass.

![Image of Himalayas with a mountain range]
What variables do you know?
• You know the bearing to the summit
• You know the difference between your elevation and that of the summit
• You know the angle the summit makes with the horizontal

What can you calculate?
You can calculate the horizontal distance between your location and the summit. Since the bearing is known, your location is known.

3. In the example illustrated below, a stream erodes less resistant fault gauge leaving an exposed fault scarp on the distant bank. You are unable to traverse the stream or make your way to the top of the exposure. Using your Brunton compass, you stand on the left edge of the stream and measure the angle (a) formed by the top of the cliff and the horizontal. You walk to the left 175 feet and measure angle (b). Angle a measure 31° and angle b, 19°. How can you determine the cliff height? What is the width of the stream?
4. The three point problem uses elevations measured at three points on a stratigraphic surface to determine the strike and dip of that surface. The elevations and locations of these points can be measured at the surface or, more likely, in the borehole. In the following problem, you have data from three boreholes (located in the map below) indicating subsea depths to the top of the Oriskany Sandstone as shown.

General overview of the 3 point problem
$\sigma = N56W$
Now, how can you determine the dip?

Measure length of line 3. Given this length and the drop in elevation you can figure the dip $\delta$ directly -

$$\tan(\delta) = \frac{300}{2080} \Rightarrow$$

$$\delta = a \tan \left[ \frac{300}{2080} \right]$$

$$\delta = 8.2^\circ$$

1. The $180^\circ$ rule: $A + B + C = 180$

2. $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$ Sine rule

3. $a^2 = b^2 + c^2 - 2bc \cos(A)$ Cosine rule

$$A = \cos^{-1} \left[ \frac{b^2 + c^2 - a^2}{2bc} \right]$$
Problems associated with triangles that do not contain a right angle can be broken down into 6 cases.