Environmental and Exploration Geophysics II

Don’t forget to visit the web site for slides and other info - http://www.geo.wvu.edu/~wilson/geol554/lect3/lec3.pdf

**Time-distance relationships**

**Ray-tracing**

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**Different travel paths**

- **P-wave**
  - Direct
  - Reflection

- **Critical Refraction**
  - Critical Angle

- **Diffraction**

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Reflection events in the shot record are completely different from those in the stack section.

Reflection time distance curve in basic hyperbolic form:

\[ t^2 = \frac{x^2}{V_i^2} + \frac{4h_i^2}{V_i^2} \]
\[ \sqrt{V_i^2 t^2} = x^2 + 4h_i^2 \]
\[ \frac{V_i^2 t^2}{4h_i^2} = \frac{x^2}{4h_i^2} + 1 \]
\[ \frac{\sqrt{V_i^2 t^2}}{4h_i} - \frac{x^2}{4h_i^2} = 1 \text{ hyperbolic form} \]
Some basic math for general perspective

General equation of a hyperbola:

\[ \frac{(y - y_0)^2}{a^2} - \frac{(x - x_0)^2}{b^2} = 1 \]

Location of the apex in time

For the reflection event:

\[ \frac{V_1^2 \tau^2}{4h_1^2} - \frac{x^2}{4h_2^2} = 1 \]

- \( y = t \), \( a = 2h_0 \)
- \( y_0 = x_0 = 0 \)
- \( b = 2h_1 \)
As time goes by, reflection events approach start to come in linearly with time. They approach the asymptotes of the hyperbola.

The direct arrival has the relationship of an asymptote to the arrival times of the reflection event.
From the basic time-distance relationship

\[ t^2 = \frac{x^2}{V_1^2} + \frac{4h^2}{V_1^2} \]

When \( x = 0 \),

\[ t^2 = \frac{4h^2}{V_1^2} \text{ or } t = \frac{2h}{V_1} \]

which is the time intercept.

When \( x \) becomes very large with respect to the thickness of the reflecting layer, the \( x^2/V_1^2 \) term becomes much larger than the \( 4h^2/V_1^2 \) term so that

\[ t^2 \approx \frac{x^2}{V_1^2} \text{ or } t \approx \frac{x}{V_1} \]
When we bang on the ground, the Earth speaks back in a variety of ways

This time-distance record shows everything coming in with different shapes, sometimes almost at the same time and sometimes earlier, sometimes later. A real mess!

The shot record does not portray subsurface structure. The differences in travel times are associated with differences in source-receiver offset.
The geology of the area is nearly flat lying.

The single layer **refraction time-distance relationship** - but first -

Snell’s law

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]

where \( n \) is the index of refraction

\[ n = \frac{c}{v} = \frac{\text{velocity of light in vacuum}}{\text{velocity of light in medium}} \]
\[ n_1 \sin \theta_1 = n_2 \sin \theta_2; \quad \frac{c}{V_1} \sin \theta_1 = \frac{c}{V_2} \sin \theta_2 \]

The c's cancel out and we have ...

One of our assumptions -

Assume \( V_1 < V_2 < V_3 \)
because \( \sin(\pi/2) = 1 \)
Critical Refraction

\[ \sin \theta_1 = \frac{v_1}{v_2}, \quad \theta_c = \text{critical angle}. \]

What is \( \theta_c \) as a function of \( x \)?

With the direct arrival and the single layer reflection, we travel from point A to B with constant velocity.

The critical refraction problem is still a simple distance over velocity function, but the velocity changes on us.

\[ \text{Time} = \frac{\text{distance travelled}}{\text{velocity of travel}}. \]
Isolating travel paths based on velocity

\[ t = \frac{2l_1}{V_1} + \frac{l_2}{V_2} \]

What's \( l_1 \) if \( l_2 \)?

The slant path length

\[ h_i = \frac{1}{\cos \Theta} \Rightarrow \gamma_i = \frac{h_i}{\cos \Theta} \]
The path along the interface

\[ X = X_{SP} + l_2 - X_{SP} - \frac{h}{V_1} \cos \theta_c \]

\[ X_{SP} = ? \]

Sum them together

\[ t = \frac{2h_1}{V_1} + \frac{l_2}{V_2} \]

\[ \tan \theta_c = \frac{X_{SP}}{h} \]

\[ X_{SP} = h \tan \theta_c \]

\[ t = \frac{2h_1}{V_1 \cos \theta_c} + \frac{X - 2h_1 \tan \theta_c}{V_2} \]

What kind of curve is this?
Variables and constants

\[ t = \frac{x}{v} + \frac{2h_m}{v_c \cos \theta_c} - \frac{2h_m \tan \theta_c}{v_c} \]

\[ f(x) = \frac{1}{v} \text{ all terms are constant} \]

\[ t = mx + b \]

\[ \text{slope } m = \frac{1}{v_c} \]

\[ \text{intercept } b = \frac{2h_m}{v_c \cos \theta_c} - \frac{2h_m \tan \theta_c}{v_c} \]

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Which of the two lines have the correct relationship of the critical refraction to the direct arrival?

![Diagram showing two lines with labels for time, x - source receiver offset distance, and Trig simplifications.](image)
Basic properties of the critical refraction

\[ \sin(\theta) = \frac{V_1}{V_2} \]

\[ \sqrt{1 - \sin^2 \theta} = \frac{V_1^2 - V_2^2}{V_1 V_2} \]

The single layer refraction

Trig transformations

\[ t = \frac{x}{V_2} + \frac{2h_1}{V_1 \cos \theta_c} - \frac{2h_1 \tan \theta_c}{V_2} \]

The critical or minimum refraction distance

The refraction time-intercept
Snell's law gives us an easy way to look trig functions in the critical triangle.
Going through the algebraic substitution the constants in our earlier expression are represented in terms of velocity instead of critical angle.

\[
\frac{X}{V_2^2} + 2h \left[ \frac{V_1^2}{V_2^2} - \frac{V_2^2}{V_1^2} \right] = \frac{X}{V_2^2} \left[ \frac{V_1^2 - V_2^2}{V_2^2} \right] = \left( \frac{1}{V_2^2} \right)X + \frac{2h}{V_2} \sqrt{V_1^2 - V_2^2} = mX + b
\]

The critical or minimum distance
Determining the critical distance

Critical reflections are not present at source-receiver distances less than the critical distance or minimum distance.

Putting all these events together in the time-distance plot reveals useful information content in the shot record.
Basic time-distance relationships for the refracted wave - Single horizontal interface

1) straight line
2) Refraction and reflection arrivals coincide at one offset ($x_{cross}$).
3) Refraction arrivals follow a straight line with
4) slope $1/V_2$, where
5) $1/V_2$ is less than $1/V_1$

Questions about the shot record

Given that the geophone interval is 10 feet determine the velocities of events A, B and C
Problem 2.6  \( V_p = \sqrt{\frac{E}{\rho} \frac{(1-\mu)}{(1-2\mu)(1+\mu)}} \)

Density range 2 to 3; Y, 0.12 to 1.1 \((x10^{11}N/m^2)\); \(\mu\), 0.04 to 0.3. Which have the greatest influence on \(V_p\). Use Table 1 (see H:Drive also see table linked on class web page) to help you assess this problem.

- How will you handle this problem?

Get together in groups and talk about this. Describe your approach and hand in. 2.6 will be due next Monday and should – from here out – be worked independently.

For next time

- Continue your reading of Chapter 2, pages 65 -77 (Chapter 3) and pages 149 - 164 (Chapter 4).
- Hand in problems 2.3 and 2.6 (next Monday)
- Look over problems 2.7, 2.12 for Monday
- Look over the absorption problem handed out today.
- Bring your questions to class next Monday.