Mathematical operation used to compute the synthetic seismogram

The convolution integral

\[ C(t) = \int_{-\infty}^{\infty} a(\tau) b(t-\tau) d\tau \]

Seismic Analog

\[ S(t) = \int_{-\infty}^{\infty} w(\tau) r(t-\tau) d\tau \]

where \( S \) is the seismic signal or trace, \( w \) is the seismic wavelet, and \( r \) is the reflectivity sequence.
Tom Wilson, Department of Geology and Geography

The output is a superposition of reflections from all acoustic interfaces and the convolution integral is a statement of the superposition principle.

Convolutional model \( S(t) = \int_{-\infty}^{\infty} w(\tau)r(t-\tau)\,d\tau \)
Discrete form of the Convolution Integral

\[ S_t = \sum_{\tau=0}^{l} w_{\tau} r_{t-\tau} \]

As defined by this equation, the process of convolution consists of 4 simple mathematical operations:

1) Folding
2) Shifting
3) Multiplication
4) Summation

Seismic Trace Attributes

Complex numbers

Complex number

\[ C = x_r + iy_i \]

Real \hspace{1cm} \text{imaginary}

also known as In-phase & Quadrature
Given the in-phase and quadrature components, it is easy to calculate the amplitude and phase or vice versa.
Seismic Data

The seismic trace is the “real” or in-phase component of the complex trace

How do we find the quadrature component?

The time-domain Hilbert Transform

In the time domain the Hilbert Transform consists of a series of values that are asymmetrical in shape: positive to one side and negative to the other. Values in the series are located at odd sample points relative to the middle of the series and diminish in magnitude with odd divisors: 1, 3, 5, etc.

Fig. 2. Normalized Hilbert time-domain operator truncated to 19 points.
View of Hilbert transform operator in relation to the samples in a seismic trace

**Figure 1.6**

**The Hilbert transform**

In the time domain: \( d^H(t) = H(t) \cdot d(t) \)

In the frequency domain: \( d^H(t) = F^{-1} \{ i \cdot F[d(t)] \} \)

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**Computing the Quadrature Trace**

**Time Series Approach**

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<th>Time</th>
<th>Real Seismic Trace</th>
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<th>Quadrature Seismic Trace</th>
<th>Reflection Strength</th>
<th>Instantaneous Phase</th>
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Recall Frequency Domain versus Time Domain Relationships

Amplitude spectrum

Phase spectrum

Individual frequency components

Time-domain wavelets

Fourier Transform of a time series

Zero Phase  Minimum Phase
The seismic response is a “real” time series.

This is its amplitude spectrum.

The Fourier Transform of a real function, like a seismic trace, is complex, i.e., it has real and imaginary parts.

The real part is even.

The imaginary part is odd.
Quadrature from the Fourier Transform

Frequency domain approach

Seismic response of the channel
Computer exercise: generating attributes and evaluating their ability to enhance the view of the channel observed in the Gulf Coast 3D volume.