Geology 659 - Quantitative Methods

Matrix methods (cont.)

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A simple boundary value problem in seismology illustrates a matrix algebra application.

\[ P_i = P_R + P_T \]

Boundary condition

An identical boundary condition holds for particle velocity

\[ v_i = v_R + v_T \]
Layer 1 has velocity $V_1$, density $\rho_1$, and impedance $Z_1 = \rho_1 V_1$.

Layer 2 has velocity $V_2$, density $\rho_2$, and impedance $Z_2 = \rho_2 V_2$.

We have the additional relationship between $v$ and $P$ stating that $P = vZ$ or $z = P/Z$.

This allows us to express both boundary conditions entirely in terms of $P$ or $V$. 
If we divide both sides of the first boundary condition by $P_i$, we get

$$1 + \frac{P_R}{P_i} = \frac{P_T}{P_i}.$$ 

$R$ is defined as $\frac{P_R}{P_i}$; $R$ is the reflection coefficient.

$T$ is defined as $\frac{P_T}{P_i}$; $T$ is the transmission coefficient.

With substitution and some rearrangement we get two additional equations.

$$R - T = -1$$

$$\frac{R}{Z_1} + \frac{T}{Z_2} = \frac{1}{Z_1}$$
Equivalently, we can write this as

\[
\begin{align*}
T - R &= 1 \\
\frac{T}{Z_2} + \frac{R}{Z_1} &= \frac{1}{Z_1}
\end{align*}
\]

We can write this set of equations in the following matrix form

\[
\begin{pmatrix}
1 & -1 \\
\frac{1}{Z_2} & \frac{1}{Z_1}
\end{pmatrix}
\begin{pmatrix}
T \\
R
\end{pmatrix}
= 
\begin{pmatrix}
1 \\
\frac{1}{Z_1}
\end{pmatrix}
\]

Which can be solved for T and R using the matrix inversion approach discussed in class on Tuesday.
In order to solve this problem we must take the inverse of the coefficient matrix.

\[
\begin{pmatrix}
T \\
R \\
\end{pmatrix} = \begin{pmatrix}
1 & -1 \\
1 & 1 \\
\frac{1}{Z_2} & \frac{1}{Z_1} \\
\end{pmatrix}^{-1} \begin{pmatrix}
1 \\
1 \\
\frac{1}{Z_1} \\
\end{pmatrix}
\]

For a simple 2 x 2 matrix the inverse is defined as

\[
\begin{pmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22} \\
\end{pmatrix}^{-1} = \begin{pmatrix}
a_{22} & -a_{12} \\
-a_{21} & a_{11} \\
\end{pmatrix}
\]

\[
\frac{1}{|A|}
\]

or just ....
Your results should be

\[ T = \frac{2Z_2}{Z_1 + Z_2} \]

\[ R = \frac{Z_2 - Z_1}{Z_1 + Z_2} \]
Seismic section across the edge of the Rome trough in western WV.
Simulating the seismic response

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<th>Velocity (ft/s)</th>
<th>RC</th>
<th>Wavelet Ormsby</th>
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These synthetics indicate that reflections from the top and base of a shallow reservoir interval are mixed in with reflections from surrounding intervals, but coincide with the negative cycle observed at about 0.31 seconds.
Comparing the synthetic response to that of the actual data.
Amplitude spectrum

Phase spectrum

Individual frequency components

Time-domain wavelets

Zero Phase  Minimum Phase
Physical nature of the seismic response
The output is a superposition of reflections from all acoustic interfaces
One additional topic to consider in general is wavelet deconvolution and how wavelet shape can affect geologic interpretations. ... Consider the following structural model:

Subsurface structure - North Sea
North Sea Seismic display after deconvolution. The geometrical interrelationships between reflectors are clearly portrayed.
The eigenvalue/eigenvector problems of concern to us in statistical analysis are associated with matrices of correlation coefficients.

Consider the 4 x 4 matrix on page 147.

\[
\begin{pmatrix}
1 & -0.28 \\
-0.28 & 1
\end{pmatrix}
\]

The matrix is symmetrical. The diagonal elements with value 1 represent the correlation of a sample with itself, while the remaining elements represent correlations of 1 sample to another.
The plots represent different states of correlation between two variables. The eigenvectors define the directions of maximum and minimum variance.

High correlation

Low correlation