Dynamic Analysis
Stress 2

Outline
- Review
- Principal Stress Components
- Stress Ellipse
- General Stress Equations
- Mohr Circle of Stress
- Examples
- 3D Mohr Circles
- States of Stress

Stress Components

Resolving stress on a plane
Given $\sigma_{xx}$ and $\sigma_{zz}$
What are $\sigma_n$ and $\sigma_s$ acting on given plane?

Magnitude of Normal Stress
As a function of Angle $\Theta$

$\sigma_n = \sigma \cos^2 \Theta$

Magnitude of Shear Stress
As a function of Angle $\Theta$

$\sigma_s = \sigma \frac{1}{2} \sin 2 \Theta$
Magnitude of Normal and Shear Stresses

\[ \sigma_1 \geq \sigma_2 \geq \sigma_3 \]

Stress Ellipsoid

Principal Stress Components

No shear stresses!

Principal Stress Components

\[
\begin{pmatrix}
\sigma_1 & 0 & 0 \\
0 & \sigma_2 & 0 \\
0 & 0 & \sigma_3
\end{pmatrix}
\]
Stress components

General Stress Equations

\[ \sigma_{nt} = \frac{1}{2} (\sigma_1 + \sigma_3) + \frac{1}{2}(\sigma_1 - \sigma_3) \cos 2\Theta \]  
(eq. 3.7)

\[ \sigma_s = \frac{1}{2}(\sigma_1 - \sigma_3) \sin 2\Theta \]  
(eq. 3.10)

\( \Theta \) = angle between plane and \( \sigma_3 \)
or between normal to the plane and \( \sigma_1 \)
+ counterclockwise, - clockwise

Mohr Circle of Stress

\[ \sigma_{nt} = \frac{1}{2} (\sigma_1 + \sigma_3) + \frac{1}{2}(\sigma_1 - \sigma_3) \cos 2\Theta \]
\[ \sigma_s = \frac{1}{2}(\sigma_1 - \sigma_3) \sin 2\Theta \]

Mean Stress

Devicative Stress

Common States of Stress

Triaxial

Biaxial

Uniaxial

Hydrostatic

Isotropic Stress

Anisotropic Stress

Trace of plane \( P \)

Weakening or closing strain

Dislocation