Dynamic Analysis
Stress 1

Reading: Chapter 4

Outline

• Force, vectors
• Units
• Normal, shear components
• Pressure
• Lithostatic Stress
• Stress Tensor
• Resolving stress on a plane

Strain = deformation
Stress = cause of deformation

Force

• What is it?
  \( F = \text{Mass} \cdot \text{acceleration} \)
• Units?
  \( \text{kg} \cdot \text{m/s}^2 \)
  
  Newtons
• How much is a Newton?
  1 Newton is about the force of gravity acting on a small apple.

Force is a vector

Strain = deformation
Stress = cause of deformation

Does applied force alone control deformation?
Stress

Stress = force / area
Units = Pascals

= N/m²
= kg/(m · s²)

Force= 50 kg * 9.8 m/s²
Area = 1 cm² = 0.0001 m²
Stress = 4,900,000 Pascals

Stress vs. Pressure

• Units of pressure = force/area
• 33 psi = 33 pounds per square inch

What is the difference?
• Pressure - Scalar quantity
• Stress - Tensor quantity

How much is 1 Pascal?

• 1 Pa = 0.00014 psi
• Car tire ~200,000 Pa
• 1 Megapascal (1 MPa) = 1 million Pa
  = 1 x 10⁶ Pa

Calculate State of Stress at 1000 m depth

Lithostatic Stress

• Similar to hydrostatic pressure
• Magnitude of stress components is the same in all directions
Stress Tensor

On-In convention:
- ON the x-plane
- IN the y-direction

Stress Tensor

\[ \begin{pmatrix}
\sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\
\sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\
\sigma_{zx} & \sigma_{zy} & \sigma_{zz}
\end{pmatrix} \]

Planes are named for the coordinate that is perpendicular to them

2D Stress Tensor

\[ \begin{pmatrix}
\sigma_{xx} & \sigma_{xz} \\
\sigma_{zx} & \sigma_{zz}
\end{pmatrix} \]

Sign Conventions

- Compression (+)
- Tension (-)
- Right Lateral Shear (+)
- Left Lateral Shear (-)

Resolving stress on a plane

Given \( \sigma_{xx} \) and \( \sigma_{zz} \)
What are \( \sigma_n \) and \( \sigma_s \) acting on a given plane?
Resolving stress on a plane

Must first correct for the change in the area

Then, you can resolve the vectors into normal and shear components

Steps

1. Sketch the geometry of the problem
2. Find the magnitude of the stress components along the coordinate axes
3. Add the two vectors
4. Resolve the resultant into normal and shear components

Example

σ\textsubscript{xx}=45 \text{ Mpa}
σ\textsubscript{zz}=26 \text{ Mpa}
x- \text{ north}
z- \text{ up}

What are the normal and shear stresses acting on a fault oriented 090/40\textdegree S?

Step 2: Find Magnitude of stress components along x and z

a= 90-\text{ dip} = 50 \text{ deg.}
σ\textsubscript{xx}= σ\textsubscript{xx} Cos a
= 45 \text{ MPa} \ Cos 50
= 28.9 \text{ Mpa}

σ\textsubscript{zz}= σ\textsubscript{zz} Sin a
= 26 \text{ MPa} \ Sin 50
= 19.9 \text{ Mpa}

Step 3: Add the two vectors

σ\textsubscript{f}^2= σ\textsubscript{fx}^2 + σ\textsubscript{fz}^2
σ\textsubscript{f}= sqrt{(28.9 \text{ Mpa}^2 + 19.9 \text{ Mpa}^2)} \text{ Mpa}
σ\textsubscript{f}= 35.1 \text{ Mpa}

b= arctan (σ\textsubscript{fz}/σ\textsubscript{fx})
b= 34.5 \text{ deg.}
Step 4: Resolve \( \sigma_f \) into normal and shear components

\[
\begin{align*}
  c &= a-b = 50 - 34.5 = 15.4 \\
  \sigma_n &= \sigma_f \cos c \\
  &= 35.1 \text{ Mpa} \cos 15.4 \\
  &= 33.8 \text{ Mpa} \\
  \sigma_s &= \sigma_f \sin c \\
  &= 35.1 \text{ Mpa} \sin 15.4 \\
  &= 9.32 \text{ MPa}
\end{align*}
\]

Magnitude of Normal Force and Normal Stress As a function of Angle \( \Theta \)

\[
\sigma_n = \sigma \cos^2 \Theta
\]

Magnitude of Shear Force and Shear Stress As a function of Angle \( \Theta \)

\[
\sigma_s = \frac{1}{2} \sin 2\Theta
\]

Principal Stress Components

You always can find an orientation of the reference cube where there won’t be any resolved shear stresses

Stress Ellipse- Normal Stress

Maximum Normal Stress

Minimum Normal Stress
Stress components

Magnitude of Normal and Shear Stresses

Principal Stress Components
(All shear stresses = 0)

Greatest

Intermediate

Least

Stress Ellipsoid

\[ \sigma_1 \geq \sigma_2 \geq \sigma_3 \]
Resolving stress on a plane

Given $\sigma_{xx}$ and $\sigma_{zz}$.

What are $\sigma_n$ and $\sigma_s$ acting on a given plane?

General Stress Equations

$\sigma_n = \frac{1}{2} (\sigma_1 + \sigma_3) + \frac{1}{2} (\sigma_1 - \sigma_3) \cos 2\Theta$

$\sigma_s = \frac{1}{2} (\sigma_1 - \sigma_3) \sin 2\Theta$

$\Theta$ = angle between plane and $\sigma_3$
or between normal to the plane and $\sigma_1$

Take Home Ideas

- Stress describes force per unit area acting on a solid body in all directions.
- Stress is made of Shear and Normal components.
- Lithostatic Stress increases at about 25 MPa/km.
- The Principal Stresses ($\sigma_1, \sigma_2, \sigma_3$) are the greatest, intermediate and least Normal stresses.